Sismo Learning

LESSON NOTE

GCSE Maths || Line segments and Midpoints

Coordinate geometry || Foundation and Higher

By the end of this lesson you will be able to

1. Explain the difference between a line and a line segment.
2. Find the midpoint of a line segment from the coordinates of its endpoints.
3. Find a missing endpoint when you know one endpoint and the midpoint.
4. Find the length of a line segment using Pythagoras' theorem.
5. Combine these skills in exam style questions.

Key words

Line	Straight, with no end in either direction.
Line segment	The part of a line between two endpoints.
Endpoint	A point at either end of a line segment.
Midpoint	The point exactly halfway along a line segment.
Coordinates	A pair (x, y) that gives the position of a point.
Length	The distance from one endpoint to the other.

Lesson map

STARTER	CONCEPT 1	CONCEPT 2	CONCEPT 3	FINISH
Lines and line segments	Midpoint of a line segment	Finding a missing endpoint	Length of a line segment	Summary and answer key

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STARTER

Lines and line segments

Before we calculate anything, we need to be clear about what we are working with.

Line

A line goes on forever in both directions. We draw arrows at both ends to show this. A line has no fixed length.



Line segment

A line segment has two endpoints. It has a fixed length, and it has exactly one midpoint. We name it by its endpoints, for example AB.



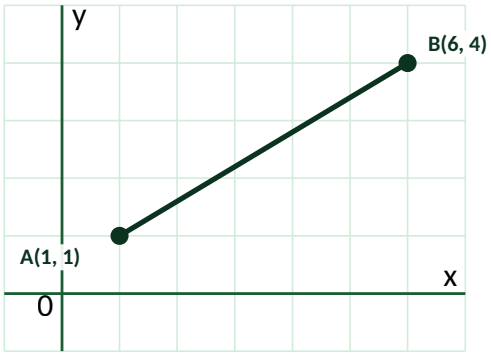
Line segments on a coordinate grid

When the endpoints of a line segment are plotted on a grid, we can describe them with coordinates. The first number is the x coordinate (across) and the second is the y coordinate (up or down).

For the segment shown, A = (1, 1) and B = (6, 4). Using only these two pairs of numbers we can work out:

- where the middle of AB is (the **midpoint**),
- where AB would end if we only knew one end and the middle,
- how long AB is (its **length**).

> **Remember:** always write coordinates as (x, y). Across first, then up or down.



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CONCEPT 1

Midpoint of a line segment

The midpoint is exactly halfway between the two endpoints, so its coordinates are halfway between theirs.

Key formula

For A(x_1, y_1) and B(x_2, y_2):

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

In words: **add the x coordinates and halve, add the y coordinates and halve.** The midpoint is the mean of the endpoints.

Watch out

- We **add** the coordinates for a midpoint. We do not subtract.
- Keep the x values together and the y values together.
- Put negative numbers in brackets: 3 + (−5).
- The midpoint can have fraction or decimal coordinates.

Worked example 1

[2 marks]

Find the midpoint of the line segment AB, where A = (2, 4) and B = (8, 10).

- **Step 1: Label the coordinates**

  A = (x_1, y_1) = (2, 4)

  B = (x_2, y_2) = (8, 10)

- **Step 2: Add the x coordinates and halve**

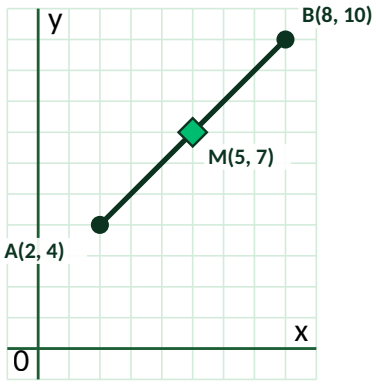
  $$x = \frac{x_1 + x_2}{2} = \frac{2 + 8}{2} = \frac{10}{2} = 5$$

- **Step 3: Add the y coordinates and halve**

  $$y = \frac{y_1 + y_2}{2} = \frac{4 + 10}{2} = \frac{14}{2} = 7$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (5, 7)



ANSWER **Midpoint = (5, 7)**

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Worked example 2

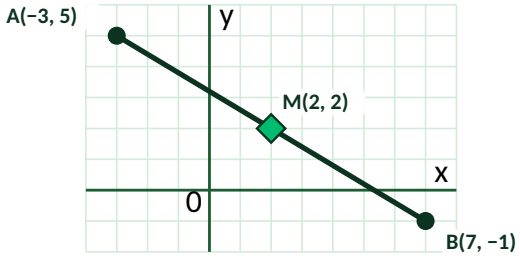
[2 marks]

Find the midpoint of the line segment AB, where A = (−3, 5) and B = (7, −1).

- **Step 1: Label the coordinates**

  A = (x_1, y_1) = (−3, 5)

  B = (x_2, y_2) = (7, −1)



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{-3 + 7}{2} = \frac{4}{2} = 2$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{5 + (-1)}{2} = \frac{4}{2} = 2$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (2, 2)

ANSWER **Midpoint = (2, 2)**

Worked example 3

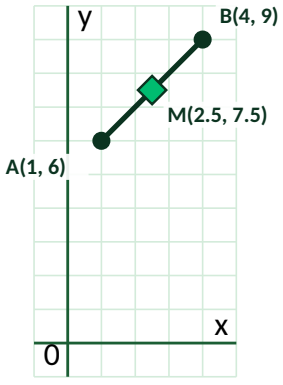
[2 marks]

Find the midpoint of the line segment AB, where A = (1, 6) and B = (4, 9).

- **Step 1: Label the coordinates**

  A = (x_1, y_1) = (1, 6)

  B = (x_2, y_2) = (4, 9)



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{1 + 4}{2} = \frac{5}{2} = 2.5$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{6 + 9}{2} = \frac{15}{2} = 7.5$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = $(\frac{5}{2}, \frac{15}{2})$

ANSWER **Midpoint = $(\frac{5}{2}, \frac{15}{2})$**

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Worked example 4

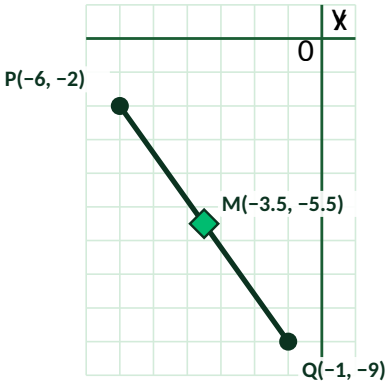
[2 marks]

Find the midpoint of the line segment PQ, where P = (−6, −2) and Q = (−1, −9).

- **Step 1: Label the coordinates**

  $P = (x_1, y_1) = (-6, -2)$

  $Q = (x_2, y_2) = (-1, -9)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{-6 + (-1)}{2} = -\frac{7}{2} = -3.5$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{-2 + (-9)}{2} = -\frac{11}{2} = -5.5$$

- **Step 4: Write the midpoint as coordinates**

$$\text{Midpoint} = \left(-\frac{7}{2}, -\frac{11}{2}\right)$$

ANSWER **Midpoint = $\left(-\frac{7}{2}, -\frac{11}{2}\right)$**

Worked example 5

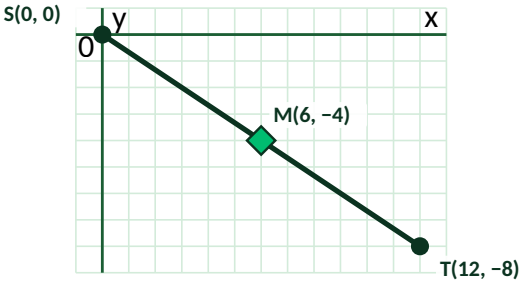
[2 marks]

On a map, a school S is at (0, 0) and a train station T is at (12, −8). A bus stop is placed exactly halfway between them. Find the coordinates of the bus stop.

- **Step 1: Label the coordinates**

  $S = (x_1, y_1) = (0, 0)$

  $T = (x_2, y_2) = (12, -8)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{0 + 12}{2} = \frac{12}{2} = 6$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{0 + (-8)}{2} = \frac{-8}{2} = -4$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (6, −4)

ANSWER **Midpoint = (6, −4)**

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Worked example 6

[2 marks]

A = (a, 2b) and B = (3a, 6b). Find the midpoint of AB in terms of a and b.

- **Step 1: Label the coordinates**

  $x_1 = a$, $y_1 = 2b$, $x_2 = 3a$, $y_2 = 6b$

- **Step 2: Add the x coordinates and halve**

  $$x = \frac{a + 3a}{2} = \frac{4a}{2} = 2a$$

- **Step 3: Add the y coordinates and halve**

  $$y = \frac{2b + 6b}{2} = \frac{8b}{2} = 4b$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (2a, 4b)

ANSWER **Midpoint = (2a, 4b)**

Practice: Concept 1

Find the midpoint of each line segment AB. Show your working.

1. A = (4, 2) and B = (10, 8) [2]
2. A = (−2, 3) and B = (6, −5) [2]
3. A = (0, −7) and B = (−8, 1) [2]
4. A = (3, 4) and B = (8, 11) [2]
5. A = (−5, −6) and B = (2, −1) [2]
6. A = (2p, q) and B = (6p, 5q). Give your answer in terms of p and q. [2]

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CONCEPT 2

Finding a missing endpoint

If you know one end and the middle, you can find the other end.

The idea

The midpoint M is halfway along AB. So the journey from A to M is **exactly the same** as the journey from M to B.

B = M + (the step from A to M)

Find how far you move in x and in y to get from A to M, then make the same move again from M.

Watch out

- The step can be negative. That means moving left or down.
- Check your answer: the midpoint of A and your B must give back M.
- A common mistake is to halve again. The missing end is further away, not closer.
- In a circle, the centre is the midpoint of any diameter.

Worked example 7

[2 marks]

M = (4, 5) is the midpoint of the line segment AB. A = (1, 3). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 4 − 1 = 3

  y step = 5 − 3 = 2

- **Step 2: Take the same step again from M**

  x = 4 + 3 = 7

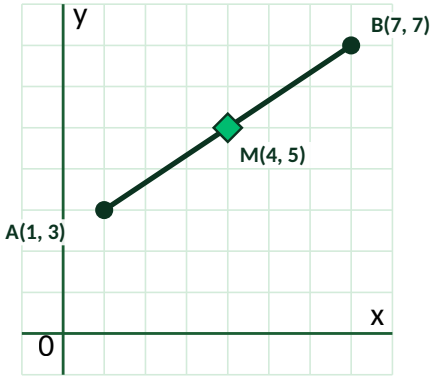
  y = 5 + 2 = 7

- **Step 3: Check with the midpoint formula**

  $\frac{1+7}{2} = \frac{8}{2} = 4$ ✓ $\frac{3+7}{2} = \frac{10}{2} = 5$ ✓

- **Step 4: Write the coordinates of B**

  B = (7, 7)



ANSWER B = (7, 7)

Worked example 8

[2 marks]

M = (1, 2) is the midpoint of the line segment AB. A = (−2, 6). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 1 − (−2) = 3

  y step = 2 − 6 = −4

- **Step 2: Take the same step again from M**

  x = 1 + 3 = 4

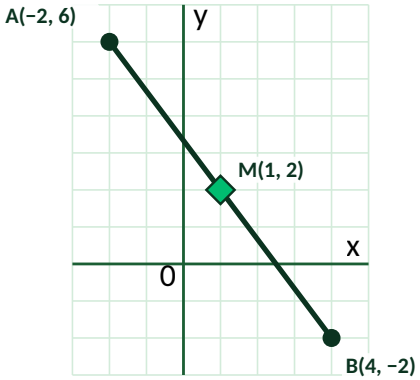
  y = 2 + (−4) = −2

- **Step 3: Check with the midpoint formula**

  $\frac{-2+4}{2} = \frac{2}{2} = 1$ ✓ $\quad \frac{6+(-2)}{2} = \frac{4}{2} = 2$ ✓

- **Step 4: Write the coordinates of B**

  B = (4, −2)



ANSWER B = (4, −2)

Worked example 9

[2 marks]

M = (0, 0) is the midpoint of the line segment AB. A = (5, −1). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 0 − 5 = −5

  y step = 0 − (−1) = 1

- **Step 2: Take the same step again from M**

  x = 0 + (−5) = −5

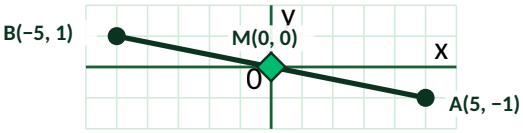
  y = 0 + 1 = 1

- **Step 3: Check with the midpoint formula**

  $\frac{5+(-5)}{2} = \frac{0}{2} = 0$ ✓ $\quad \frac{-1+1}{2} = \frac{0}{2} = 0$ ✓

- **Step 4: Write the coordinates of B**

  B = (−5, 1)

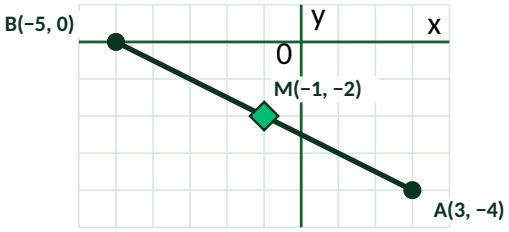


ANSWER B = (−5, 1)

Worked example 10

[2 marks]

M = (−1, −2) is the midpoint of the line segment AB. A = (3, −4). Find the coordinates of B.



- **Step 1: Find the step from A to M**

  x step = −1 − 3 = −4

  y step = −2 − (−4) = 2

- **Step 2: Take the same step again from M**

  x = −1 + (−4) = −5

  y = −2 + 2 = 0

- **Step 3: Check with the midpoint formula**

  $\frac{3+(-5)}{2} = \frac{-2}{2} = -1$ ✓ $\quad \frac{-4+0}{2} = \frac{-4}{2} = -2$ ✓

- **Step 4: Write the coordinates of B**

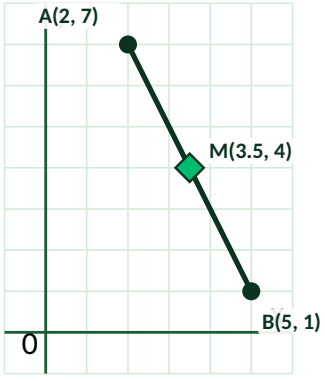
  B = (−5, 0)

ANSWER **B = (−5, 0)**

Worked example 11

[2 marks]

M = (3.5, 4) is the midpoint of the line segment AB. A = (2, 7). Find the coordinates of B.



- **Step 1: Find the step from A to M**

  x step = 3.5 − 2 = 1.5

  y step = 4 − 7 = −3

- **Step 2: Take the same step again from M**

  x = 3.5 + 1.5 = 5

  y = 4 + (−3) = 1

- **Step 3: Check with the midpoint formula**

  $\frac{2+5}{2} = \frac{7}{2} = 3.5$ ✓ $\quad \frac{7+1}{2} = \frac{8}{2} = 4$ ✓

- **Step 4: Write the coordinates of B**

  B = (5, 1)

ANSWER **B = (5, 1)**

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Worked example 12

[3 marks]

A circle has centre C = (2, −1). PQ is a diameter of the circle and P = (−3, 4). Find the coordinates of Q.

- **Step 1: Find the step from P to C**

  x step = 2 − (−3) = 5

  y step = −1 − 4 = −5

- **Step 2: Take the same step again from C**

  x = 2 + 5 = 7

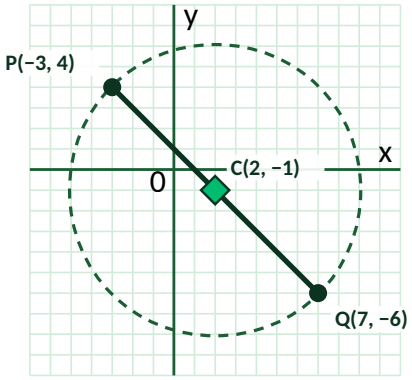
  y = −1 + (−5) = −6

- **Step 3: Check with the midpoint formula**

  $\frac{-3+7}{2} = \frac{4}{2} = 2$ ✓ $\quad \frac{4+(-6)}{2} = \frac{-2}{2} = -1$ ✓

- **Step 4: Write the coordinates of Q**

  Q = (7, −6)



ANSWER Q = (7, −6)

Practice: Concept 2

M is the midpoint of AB. Find the coordinates of B. Show your working.

7. A = (2, 1) and M = (5, 4) [2]

8. A = (−3, 4) and M = (1, 1) [2]

9. A = (6, −2) and M = (2, −5) [2]

10. A = (−4, −7) and M = (−1, 0) [2]

11. A = (1, 5) and M = (4.5, 2) [2]

12. A circle has centre (−2, 3). One end of a diameter is (1, 7). Find the other end of the diameter. [3]

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CONCEPT 3

Length of a line segment

Every sloping line segment is the longest side of a right angled triangle, so we use Pythagoras' theorem.

Key formula

Draw the horizontal change and the vertical change to make a right angled triangle. AB is the hypotenuse.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal change squared, plus vertical change squared, then square root.

Watch out

- Subtract carefully: $4 - (-2) = 6$.
- Square negative numbers in brackets: $(-8)^2 = 64$, never -64.
- A length is never negative.
- Keep the full calculator value and round only at the end.

Worked example 13

[3 marks]

Find the length of the line segment AB, where A = (1, 2) and B = (4, 6).

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 4 - 1 = 3$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 6 - 2 = 4$

- **Step 3: Use Pythagoras' theorem**

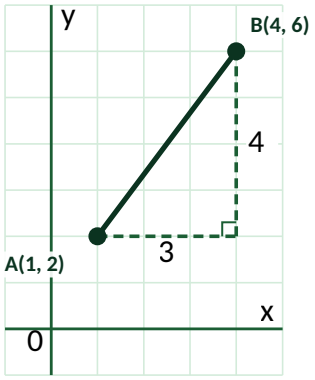
  $AB^2 = 3^2 + 4^2$

  $AB^2 = 9 + 16$

  $AB^2 = 25$

- **Step 4: Take the square root**

  $AB = \sqrt{25} = 5$



ANSWER AB = 5 units

Worked example 14

[3 marks]

Find the length of the line segment AB, where A = (−2, 3) and B = (4, −5).

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 4 - (-2) = 6$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = -5 - 3 = -8$

- **Step 3: Use Pythagoras' theorem**

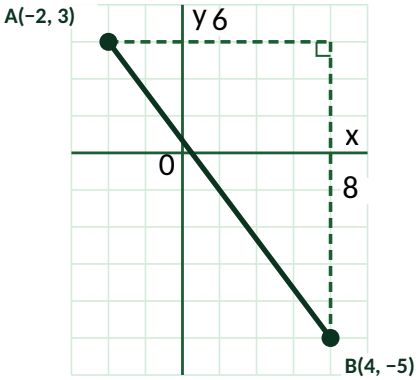
  $AB^2 = 6^2 + (-8)^2$

  $AB^2 = 36 + 64$

  $AB^2 = 100$

- **Step 4: Take the square root**

  $AB = \sqrt{100} = 10$



ANSWER AB = 10 units

Worked example 15

[3 marks]

Find the length of the line segment AB, where A = (−1, −4) and B = (4, 8).

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 4 - (-1) = 5$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 8 - (-4) = 12$

- **Step 3: Use Pythagoras' theorem**

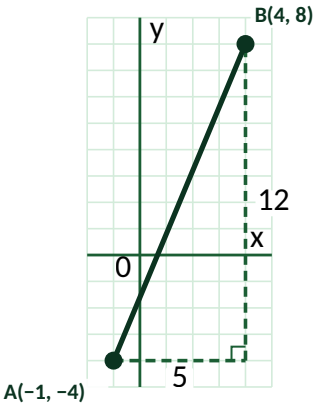
  $AB^2 = 5^2 + 12^2$

  $AB^2 = 25 + 144$

  $AB^2 = 169$

- **Step 4: Take the square root**

  $AB = \sqrt{169} = 13$



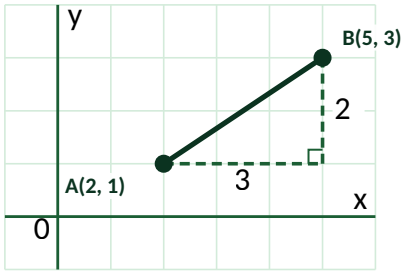
ANSWER AB = 13 units

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Worked example 16

[3 marks]

Find the length of the line segment AB, where A = (2, 1) and B = (5, 3). Give your answer to 2 decimal places.



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 5 - 2 = 3$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 3 - 1 = 2$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = 3^2 + 2^2$

  $AB^2 = 9 + 4$

  $AB^2 = 13$

- **Step 4: Take the square root**

  $AB = \sqrt{13} = 3.6055...$

- **Step 5: Round at the final step**

  AB = 3.61 (2 d.p.)

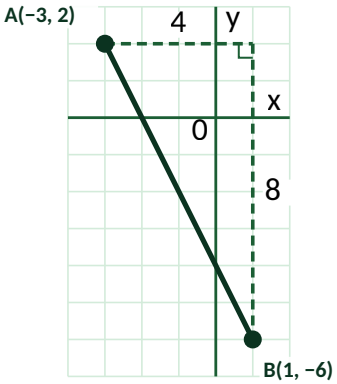
ANSWER **AB = 3.61 units (2 d.p.)**

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Worked example 17

[3 marks]

Find the length of the line segment AB, where A = (−3, 2) and B = (1, −6). Give your answer to 2 decimal places.



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 1 - (-3) = 4$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = -6 - 2 = -8$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = 4^2 + (-8)^2$

  $AB^2 = 16 + 64$

  $AB^2 = 80$

- **Step 4: Take the square root**

  $AB = \sqrt{80} = 8.9442...$

- **Step 5: Round at the final step**

  AB = 8.94 (2 d.p.)

- **Step 6: Higher tier: write it as a simplified surd**

  $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

ANSWER AB = 8.94 units (2 d.p.)

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Worked example 18

[3 marks]

P = (1, −2) and Q = (7, 6). Show that the length of PQ is 10 units.

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 7 - 1 = 6$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 6 - (-2) = 8$

- **Step 3: Use Pythagoras' theorem**

  $PQ^2 = 6^2 + 8^2$

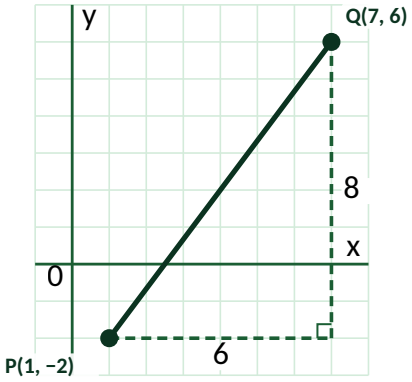
  $PQ^2 = 36 + 64$

  $PQ^2 = 100$

- **Step 4: Take the square root**

  $PQ = \sqrt{100} = 10$

  PQ = 10 as required ✓



ANSWER **PQ = 10 units (shown) ✓**

Practice: Concept 3

Find the length of each line segment AB. Where the answer is not a whole number, give it to 2 decimal places.

13. A = (2, 3) and B = (5, 7) **[3]**

14. A = (−1, 2) and B = (5, 10) **[3]**

15. A = (3, −4) and B = (−2, 8) **[3]**

16. A = (1, 1) and B = (4, 3) **[3]**

17. A = (−2, −3) and B = (3, 1) **[3]**

18. A = (−4, 1) and B = (2, 9). Show that the length of AB is 10 units. **[3]**

Exam style question

19. A = (−2, 1) and B = (6, 7).

(a) Find the coordinates of the midpoint of AB. **[2]**

(b) Find the length of AB. **[3]**

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Summary

To find	Method	Formula
Midpoint	Add and halve each coordinate	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Missing endpoint	Repeat the step from A to M	B = M + (M − A)
Length	Pythagoras' theorem	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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ANSWER KEY

Answer Summary

Final answers to Practice questions 1 to 19. Full worked solutions follow.

Q	Answer	Q	Answer
1	(7, 5)	**11**	B = (8, −1)
2	(2, −1)	**12**	Q = (−5, −1)
3	(−4, −3)	**13**	AB = 5
4	$(\frac{11}{2}, \frac{15}{2})$	**14**	AB = 10
5	$(-\frac{3}{2}, -\frac{7}{2})$	**15**	AB = 13
6	(4p, 3q)	**16**	AB = 3.61 (2 d.p.)
7	B = (8, 7)	**17**	AB = 6.40 (2 d.p.)
8	B = (5, −2)	**18**	AB = 10 (shown)
9	B = (−2, −8)	**19(a)**	(2, 4)
10	B = (2, 7)	**19(b)**	AB = 10

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ANSWER KEY

Worked solutions

Question 1

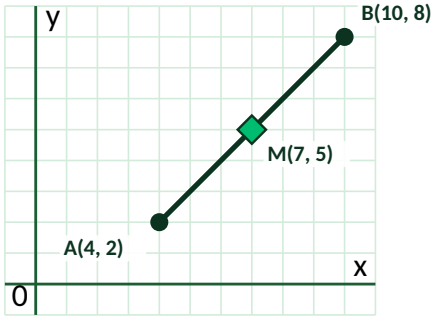
[2 marks]

Find the midpoint of the line segment AB, where A = (4, 2) and B = (10, 8).

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (4, 2)$

  $B = (x_2, y_2) = (10, 8)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{4 + 10}{2} = \frac{14}{2} = 7$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{2 + 8}{2} = \frac{10}{2} = 5$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (7, 5)

ANSWER Midpoint = (7, 5)

Question 2

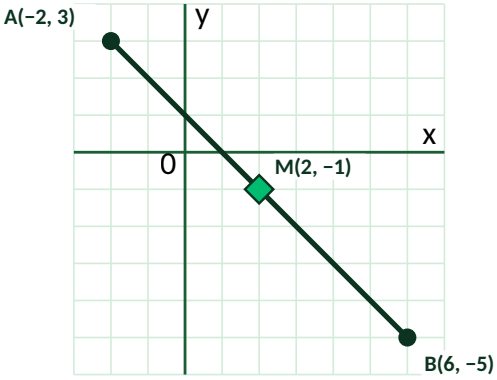
[2 marks]

Find the midpoint of the line segment AB, where A = (−2, 3) and B = (6, −5).

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (-2, 3)$

  $B = (x_2, y_2) = (6, -5)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{-2 + 6}{2} = \frac{4}{2} = 2$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{3 + (-5)}{2} = \frac{-2}{2} = -1$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (2, −1)

ANSWER Midpoint = (2, −1)

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Question 3

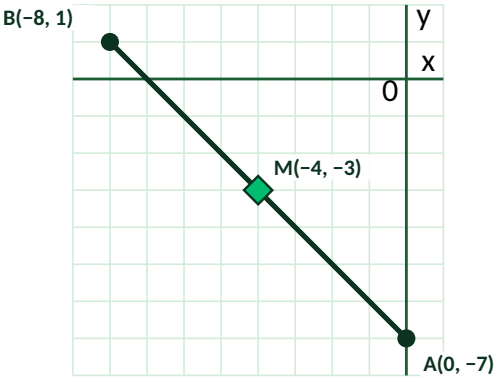
[2 marks]

Find the midpoint of the line segment AB, where A = (0, −7) and B = (−8, 1).

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (0, -7)$

  $B = (x_2, y_2) = (-8, 1)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{0 + (-8)}{2} = \frac{-8}{2} = -4$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{-7 + 1}{2} = \frac{-6}{2} = -3$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (−4, −3)

ANSWER **Midpoint = (−4, −3)**

Question 4

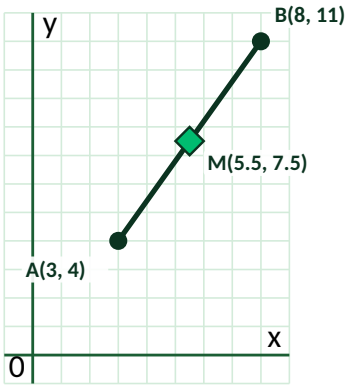
[2 marks]

Find the midpoint of the line segment AB, where A = (3, 4) and B = (8, 11).

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (3, 4)$

  $B = (x_2, y_2) = (8, 11)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{3 + 8}{2} = \frac{11}{2} = 5.5$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{4 + 11}{2} = \frac{15}{2} = 7.5$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = $(\frac{11}{2}, \frac{15}{2})$

ANSWER **Midpoint = $(\frac{11}{2}, \frac{15}{2})$**

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Question 5

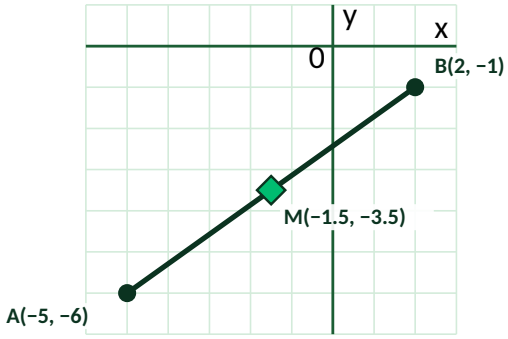
[2 marks]

Find the midpoint of the line segment AB, where A = (−5, −6) and B = (2, −1).

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (-5, -6)$

  $B = (x_2, y_2) = (2, -1)$



- **Step 2: Add the x coordinates and halve**

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 2}{2} = -\frac{3}{2} = -1.5$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{y_1 + y_2}{2} = \frac{-6 + (-1)}{2} = -\frac{7}{2} = -3.5$$

- **Step 4: Write the midpoint as coordinates**

$$\text{Midpoint} = \left(-\frac{3}{2}, -\frac{7}{2}\right)$$

ANSWER **Midpoint = $\left(-\frac{3}{2}, -\frac{7}{2}\right)$**

Question 6

[2 marks]

Find the midpoint of AB, where A = (2p, q) and B = (6p, 5q).

- **Step 1: Label the coordinates**

  $x_1 = 2p, y_1 = q, x_2 = 6p, y_2 = 5q$

- **Step 2: Add the x coordinates and halve**

$$x = \frac{2p + 6p}{2} = \frac{8p}{2} = 4p$$

- **Step 3: Add the y coordinates and halve**

$$y = \frac{q + 5q}{2} = \frac{6q}{2} = 3q$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (4p, 3q)

ANSWER **Midpoint = (4p, 3q)**

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Question 7

[2 marks]

M = (5, 4) is the midpoint of the line segment AB. A = (2, 1). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 5 − 2 = 3

  y step = 4 − 1 = 3

- **Step 2: Take the same step again from M**

  x = 5 + 3 = 8

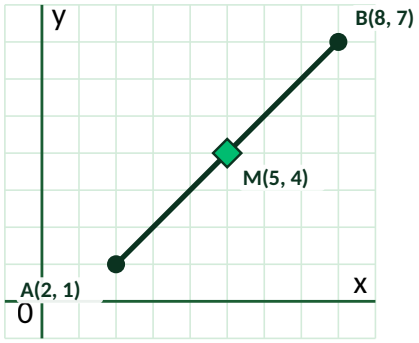
  y = 4 + 3 = 7

- **Step 3: Check with the midpoint formula**

  $\frac{2+8}{2} = \frac{10}{2} = 5$ ✓ $\quad \frac{1+7}{2} = \frac{8}{2} = 4$ ✓

- **Step 4: Write the coordinates of B**

  B = (8, 7)



ANSWER B = (8, 7)

Question 8

[2 marks]

M = (1, 1) is the midpoint of the line segment AB. A = (−3, 4). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 1 − (−3) = 4

  y step = 1 − 4 = −3

- **Step 2: Take the same step again from M**

  x = 1 + 4 = 5

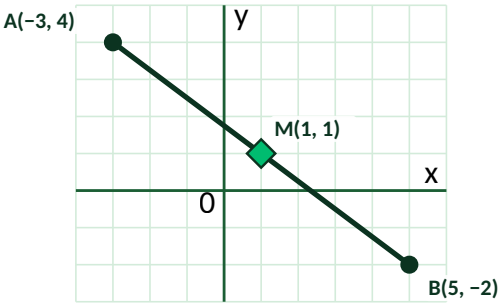
  y = 1 + (−3) = −2

- **Step 3: Check with the midpoint formula**

  $\frac{-3+5}{2} = \frac{2}{2} = 1$ ✓ $\quad \frac{4+(-2)}{2} = \frac{2}{2} = 1$ ✓

- **Step 4: Write the coordinates of B**

  B = (5, −2)



ANSWER B = (5, −2)

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Question 9

[2 marks]

M = (2, −5) is the midpoint of the line segment AB. A = (6, −2). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 2 − 6 = −4

  y step = −5 − (−2) = −3

- **Step 2: Take the same step again from M**

  x = 2 + (−4) = −2

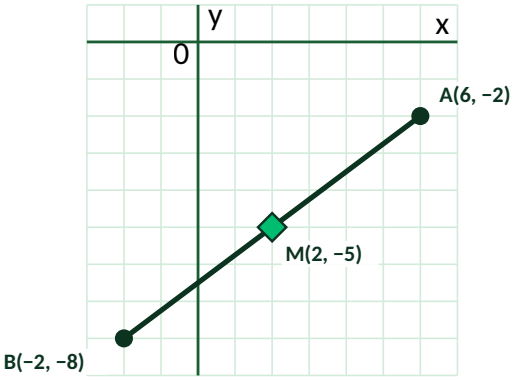
  y = −5 + (−3) = −8

- **Step 3: Check with the midpoint formula**

  $\frac{6+(-2)}{2} = \frac{4}{2} = 2$ ✓ $\quad \frac{-2+(-8)}{2} = \frac{-10}{2} = -5$ ✓

- **Step 4: Write the coordinates of B**

  B = (−2, −8)



ANSWER B = (−2, −8)

Question 10

[2 marks]

M = (−1, 0) is the midpoint of the line segment AB. A = (−4, −7). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = −1 − (−4) = 3

  y step = 0 − (−7) = 7

- **Step 2: Take the same step again from M**

  x = −1 + 3 = 2

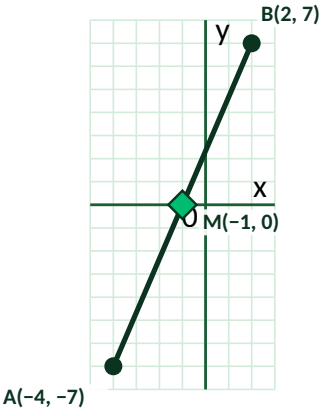
  y = 0 + 7 = 7

- **Step 3: Check with the midpoint formula**

  $\frac{-4+2}{2} = \frac{-2}{2} = -1$ ✓ $\quad \frac{-7+7}{2} = \frac{0}{2} = 0$ ✓

- **Step 4: Write the coordinates of B**

  B = (2, 7)



ANSWER B = (2, 7)

Question 11

[2 marks]

M = (4.5, 2) is the midpoint of the line segment AB. A = (1, 5). Find the coordinates of B.

- **Step 1: Find the step from A to M**

  x step = 4.5 − 1 = 3.5

  y step = 2 − 5 = −3

- **Step 2: Take the same step again from M**

  x = 4.5 + 3.5 = 8

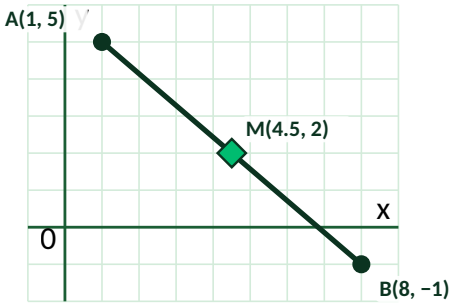
  y = 2 + (−3) = −1

- **Step 3: Check with the midpoint formula**

  $\frac{1+8}{2} = \frac{9}{2} = 4.5$ ✓ $\quad \frac{5+(-1)}{2} = \frac{4}{2} = 2$ ✓

- **Step 4: Write the coordinates of B**

  B = (8, −1)



ANSWER B = (8, −1)

Question 12

[3 marks]

A circle has centre C = (−2, 3). One end of a diameter is P = (1, 7). Find the other end, Q.

- **Step 1: Find the step from P to C**

  x step = −2 − 1 = −3

  y step = 3 − 7 = −4

- **Step 2: Take the same step again from C**

  x = −2 + (−3) = −5

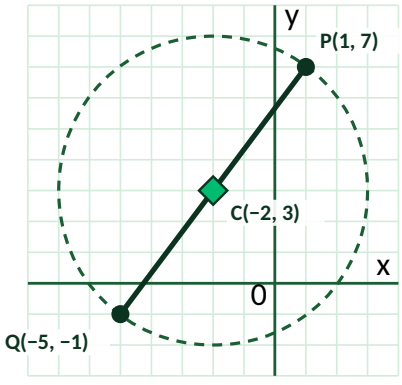
  y = 3 + (−4) = −1

- **Step 3: Check with the midpoint formula**

  $\frac{1+(-5)}{2} = \frac{-4}{2} = -2$ ✓ $\quad \frac{7+(-1)}{2} = \frac{6}{2} = 3$ ✓

- **Step 4: Write the coordinates of Q**

  Q = (−5, −1)



ANSWER Q = (−5, −1)

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Question 13

[3 marks]

Find the length of the line segment AB, where A = (2, 3) and B = (5, 7).

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 5 - 2 = 3$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 7 - 3 = 4$

- **Step 3: Use Pythagoras' theorem**

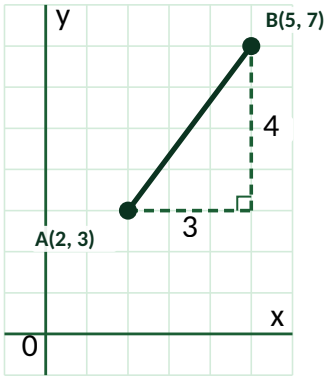
  $AB^2 = 3^2 + 4^2$

  $AB^2 = 9 + 16$

  $AB^2 = 25$

- **Step 4: Take the square root**

  $AB = \sqrt{25} = 5$



ANSWER AB = 5 units

Question 14

[3 marks]

Find the length of the line segment AB, where A = (−1, 2) and B = (5, 10).

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 5 - (-1) = 6$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 10 - 2 = 8$

- **Step 3: Use Pythagoras' theorem**

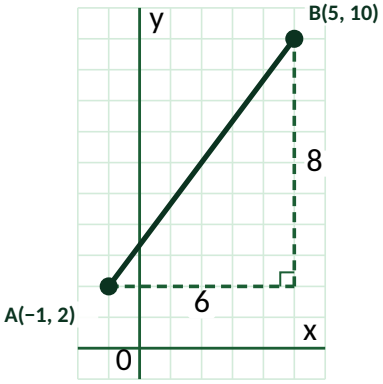
  $AB^2 = 6^2 + 8^2$

  $AB^2 = 36 + 64$

  $AB^2 = 100$

- **Step 4: Take the square root**

  $AB = \sqrt{100} = 10$



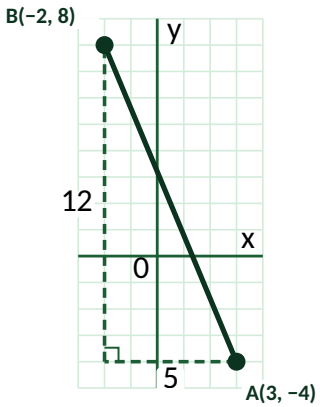
ANSWER AB = 10 units

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Question 15

[3 marks]

Find the length of the line segment AB, where A = (3, −4) and B = (−2, 8).



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = -2 - 3 = -5$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 8 - (-4) = 12$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = (-5)^2 + 12^2$

  $AB^2 = 25 + 144$

  $AB^2 = 169$

- **Step 4: Take the square root**

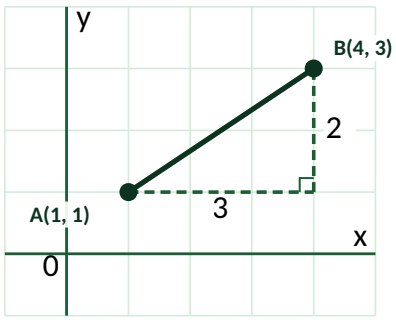
  $AB = \sqrt{169} = 13$

ANSWER **AB = 13 units**

Question 16

[3 marks]

Find the length of the line segment AB, where A = (1, 1) and B = (4, 3). Give your answer to 2 decimal places.



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 4 - 1 = 3$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 3 - 1 = 2$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = 3^2 + 2^2$

  $AB^2 = 9 + 4$

  $AB^2 = 13$

- **Step 4: Take the square root**

  $AB = \sqrt{13} = 3.6055...$

- **Step 5: Round at the final step**

  AB = 3.61 (2 d.p.)

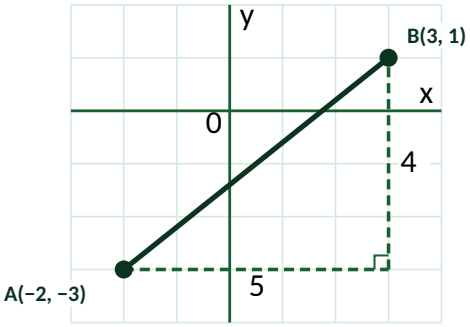
ANSWER **AB = 3.61 units (2 d.p.)**

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Question 17

[3 marks]

Find the length of the line segment AB, where A = (−2, −3) and B = (3, 1). Give your answer to 2 decimal places.



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 3 - (-2) = 5$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 1 - (-3) = 4$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = 5^2 + 4^2$

  $AB^2 = 25 + 16$

  $AB^2 = 41$

- **Step 4: Take the square root**

  $AB = \sqrt{41} = 6.4031...$

- **Step 5: Round at the final step**

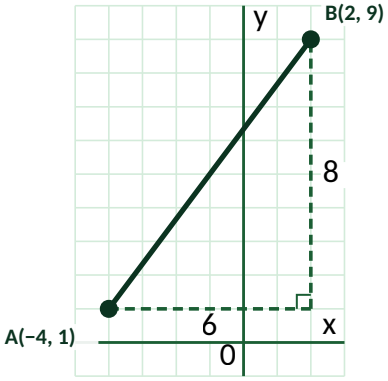
  AB = 6.40 (2 d.p.)

ANSWER AB = 6.40 units (2 d.p.)

Question 18

[3 marks]

A = (−4, 1) and B = (2, 9). Show that the length of AB is 10 units.



- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 2 - (-4) = 6$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 9 - 1 = 8$

- **Step 3: Use Pythagoras' theorem**

  $AB^2 = 6^2 + 8^2$

  $AB^2 = 36 + 64$

  $AB^2 = 100$

- **Step 4: Take the square root**

  $AB = \sqrt{100} = 10$

  AB = 10 as required ✓

ANSWER AB = 10 units (shown) ✓

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Question 19(a)

[2 marks]

A = (−2, 1) and B = (6, 7). Find the coordinates of the midpoint of AB.

- **Step 1: Label the coordinates**

  $A = (x_1, y_1) = (-2, 1)$

  $B = (x_2, y_2) = (6, 7)$

- **Step 2: Add the x coordinates and halve**

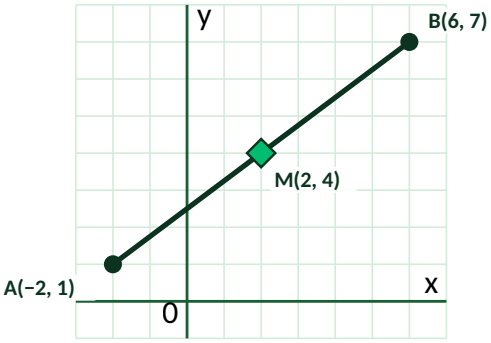
  $$x = \frac{x_1 + x_2}{2} = \frac{-2 + 6}{2} = \frac{4}{2} = 2$$

- **Step 3: Add the y coordinates and halve**

  $$y = \frac{y_1 + y_2}{2} = \frac{1 + 7}{2} = \frac{8}{2} = 4$$

- **Step 4: Write the midpoint as coordinates**

  Midpoint = (2, 4)



ANSWER Midpoint = (2, 4)

Question 19(b)

[3 marks]

A = (−2, 1) and B = (6, 7). Find the length of AB.

- **Step 1: Find the horizontal change**

  $x_2 - x_1 = 6 - (-2) = 8$

- **Step 2: Find the vertical change**

  $y_2 - y_1 = 7 - 1 = 6$

- **Step 3: Use Pythagoras' theorem**

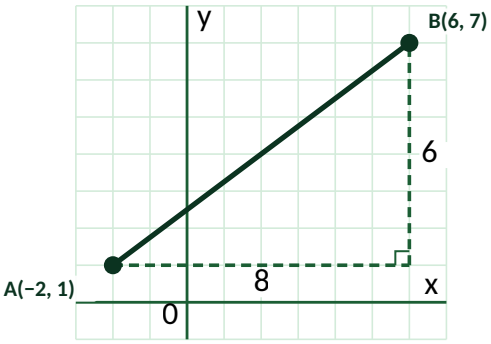
  $AB^2 = 8^2 + 6^2$

  $AB^2 = 64 + 36$

  $AB^2 = 100$

- **Step 4: Take the square root**

  $AB = \sqrt{100} = 10$



ANSWER AB = 10 units

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