Sismo Learning

YEAR 9 || LESSON NOTE

Multiplication of surds

Strengthening multiplication with surds: from $\sqrt{a} \times \sqrt{b}$ through to expanding double brackets, with every solution worked step by step.

S	I	S	M	O
See	Investigate	Solve	Master	Outperform

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Lesson at a glance

By the end of this lesson I can

- ✔ explain what a surd is and spot one
- ✔ multiply two or more surds using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- ✔ write a surd answer in its simplest form
- ✔ multiply surds that have whole number coefficients
- ✔ square and cube surds
- ✔ expand single and double brackets that contain surds

Key words

Surd: a root that cannot be written as a whole number or fraction, such as $\sqrt{2}$.
Coefficient: the number in front, such as 5 in $5\sqrt{3}$.
Square factor: a square number that divides exactly, such as 4 in 12.
Simplest form: no square factor left under the root.
Expand: multiply out a bracket.

Before you start: square numbers you must know

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144

How this note works

Each concept gives a short explanation, the key rule, a common mistake to avoid, six worked examples set out the way you would write them on paper, then six questions for you to try. The answers to every practice question are in the Answer Key at the back, fully worked.

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See and Investigate

S || SEE

What is a surd?

Some square roots are whole numbers: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{25} = 5$. Others are not. $\sqrt{2} = 1.4142135\ldots$ and the decimals never end or repeat. A root like this is called a **surd**. We leave it written as $\sqrt{2}$ because that is exact, while any decimal is only an approximation.

Root	Value	Surd?
$\sqrt{16}$	4	No, 16 is a square number
$\sqrt{3}$	1.7320508…	**Yes**
$\sqrt{36}$	6	No, 36 is a square number
$\sqrt{10}$	3.1622776…	**Yes**

I || INVESTIGATE

Spot the pattern

Use a calculator. Keep the full values on the screen.

Multiply the roots	Root of the product	What do you notice?
$\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$	$\sqrt{36} = 6$	**Same**
$\sqrt{2} \times \sqrt{3} = 2.4494897\ldots$	$\sqrt{6} = 2.4494897\ldots$	**Same**
$\sqrt{5} \times \sqrt{5} = 5$	$\sqrt{25} = 5$	**Same**

Multiplying the roots gives the same result as taking the root of the product. This gives us the most important rule of the lesson.

KEY RULES

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \sqrt{a} \times \sqrt{a} = a \qquad p\sqrt{a} \times q\sqrt{b} = pq\sqrt{ab}$$

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Concept 1 || Multiplying two surds

S || SOLVE

To multiply two surds, multiply the numbers under the root signs and keep them under one root sign. When a surd is multiplied by itself, the root sign disappears and you are left with the number underneath.

KEY RULE

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\quad$ $\sqrt{a} \times \sqrt{a} = a$

WATCH OUT

× $\sqrt{2} + \sqrt{3} = \sqrt{5}$

The rule only works for multiplication. $\sqrt{2} + \sqrt{3}$ cannot be combined into one surd.

Worked examples

WORKED EXAMPLE 1.1 — 2 marks

$\sqrt{2} \times \sqrt{3}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3}$

$= \sqrt{6}$

Step 2: Check for square factors

6 has no square factor other than 1, so $\sqrt{6}$ is already in simplest form

ANSWER $\sqrt{6}$

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WORKED EXAMPLE 1.2

2 marks

$\sqrt{5} \times \sqrt{7}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{5} \times \sqrt{7} = \sqrt{5 \times 7}$

$= \sqrt{35}$

Step 2: Check for square factors

35 has no square factor other than 1, so $\sqrt{35}$ is already in simplest form

ANSWER $\sqrt{35}$

WORKED EXAMPLE 1.3

2 marks

$\sqrt{6} \times \sqrt{6}$

Step 1: Multiply the surd by itself

$\sqrt{6} \times \sqrt{6} = \sqrt{6 \times 6}$

$= \sqrt{36}$

= 6 (a surd multiplied by itself gives the number under the root)

ANSWER 6

WORKED EXAMPLE 1.4

2 marks

$\sqrt{2} \times \sqrt{8}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8}$

$= \sqrt{16}$

Step 2: Simplify the surd

16 is a square number, so $\sqrt{16} = 4$

ANSWER 4

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WORKED EXAMPLE 1.5 — 2 marks

$\sqrt{3} \times \sqrt{12}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12}$

$= \sqrt{36}$

Step 2: Simplify the surd

36 is a square number, so $\sqrt{36} = 6$

ANSWER 6

WORKED EXAMPLE 1.6 — 2 marks

$\sqrt{5} \times \sqrt{2} \times \sqrt{3}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{5} \times \sqrt{2} \times \sqrt{3} = \sqrt{5 \times 2 \times 3}$

$= \sqrt{30}$

Step 2: Check for square factors

30 has no square factor other than 1, so $\sqrt{30}$ is already in simplest form

ANSWER $\sqrt{30}$

M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. $\sqrt{3} \times \sqrt{5}$ [2]
2. $\sqrt{7} \times \sqrt{2}$ [2]
3. $\sqrt{11} \times \sqrt{11}$ [2]
4. $\sqrt{2} \times \sqrt{18}$ [2]
5. $\sqrt{5} \times \sqrt{20}$ [2]
6. $\sqrt{2} \times \sqrt{3} \times \sqrt{7}$ [2]

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Concept 2 || Simplifying the answer

S || SOLVE

An answer such as $\sqrt{12}$ is correct, but it is not finished. A surd is in its simplest form when the number under the root has no square factor other than 1. Find the largest square number that divides into it, split the root, then square root the square number.

KEY RULE

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \qquad \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

WATCH OUT

✗ $\sqrt{12} = \sqrt{2} \times \sqrt{6}$

This is true but not helpful. Always split using the largest square factor, so one of the roots becomes a whole number.

Worked examples

WORKED EXAMPLE 2.1

2 marks

Simplify $\sqrt{12}$

Step 1: List the square numbers

4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Step 2: Find the largest square factor of 12

$12 = 4 \times 3$

Step 3: Split the root into two roots

$\sqrt{12} = \sqrt{4} \times \sqrt{3}$

Step 4: Square root the square number

$= 2 \times \sqrt{3}$

$= 2\sqrt{3}$

ANSWER $2\sqrt{3}$

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WORKED EXAMPLE 2.2

2 marks

Simplify $\sqrt{50}$

Step 1: List the square numbers

4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Step 2: Find the largest square factor of 50

$50 = 25 \times 2$

Step 3: Split the root into two roots

$\sqrt{50} = \sqrt{25} \times \sqrt{2}$

Step 4: Square root the square number

$= 5 \times \sqrt{2}$

$= 5\sqrt{2}$

ANSWER $5\sqrt{2}$

WORKED EXAMPLE 2.3

2 marks

Simplify $\sqrt{72}$

Step 1: List the square numbers

4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Step 2: Find the largest square factor of 72

$72 = 36 \times 2$

Step 3: Split the root into two roots

$\sqrt{72} = \sqrt{36} \times \sqrt{2}$

Step 4: Square root the square number

$= 6 \times \sqrt{2}$

$= 6\sqrt{2}$

ANSWER $6\sqrt{2}$

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WORKED EXAMPLE 2.4

2 marks

$\sqrt{2} \times \sqrt{6}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6}$

$= \sqrt{12}$

Step 2: Simplify the surd

$12 = 4 \times 3$ (4 is the largest square factor)

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

ANSWER $2\sqrt{3}$

WORKED EXAMPLE 2.5

2 marks

$\sqrt{3} \times \sqrt{15}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{15} = \sqrt{3 \times 15}$

$= \sqrt{45}$

Step 2: Simplify the surd

$45 = 9 \times 5$ (9 is the largest square factor)

$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

ANSWER $3\sqrt{5}$

WORKED EXAMPLE 2.6

2 marks

$\sqrt{6} \times \sqrt{8}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{6} \times \sqrt{8} = \sqrt{6 \times 8}$

$= \sqrt{48}$

Step 2: Simplify the surd

$48 = 16 \times 3$ (16 is the largest square factor)

$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

ANSWER $4\sqrt{3}$

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M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. Simplify $\sqrt{18}$ [2]
2. Simplify $\sqrt{75}$ [2]
3. $\sqrt{2} \times \sqrt{10}$ [2]
4. $\sqrt{3} \times \sqrt{6}$ [2]
5. $\sqrt{5} \times \sqrt{15}$ [2]
6. $\sqrt{10} \times \sqrt{8}$ [2]

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Concept 3 || Surds with whole number coefficients

S || SOLVE

A coefficient is the whole number in front of a surd, such as the 4 in $4\sqrt{5}$. Multiply the whole numbers together, multiply the surds together, then simplify.

KEY RULE

$$p\sqrt{a} \times q\sqrt{b} = pq\sqrt{ab}$$

WATCH OUT

× $2\sqrt{3} \times 4\sqrt{5} = 6\sqrt{15}$

The whole numbers are multiplied, not added: $2 \times 4 = 8$, so the answer is $8\sqrt{15}$.

Worked examples

WORKED EXAMPLE 3.1

2 marks

$2\sqrt{3} \times 4\sqrt{5}$

Step 1: Group the whole numbers and the surds

$2\sqrt{3} \times 4\sqrt{5} = (2 \times 4) \times (\sqrt{3} \times \sqrt{5})$

Step 2: Multiply the whole numbers

$2 \times 4 = 8$

Step 3: Multiply the surds

$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$

Step 4: Put the results together

$8 \times \sqrt{15} = 8\sqrt{15}$

ANSWER $8\sqrt{15}$

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WORKED EXAMPLE 3.2

2 marks

$3\sqrt{2} \times 5\sqrt{7}$

Step 1: Group the whole numbers and the surds

$3\sqrt{2} \times 5\sqrt{7} = (3 \times 5) \times (\sqrt{2} \times \sqrt{7})$

Step 2: Multiply the whole numbers

$3 \times 5 = 15$

Step 3: Multiply the surds

$\sqrt{2} \times \sqrt{7} = \sqrt{2 \times 7} = \sqrt{14}$

Step 4: Put the results together

$15 \times \sqrt{14} = 15\sqrt{14}$

ANSWER $15\sqrt{14}$

WORKED EXAMPLE 3.3

2 marks

$2\sqrt{5} \times 3\sqrt{5}$

Step 1: Group the whole numbers and the surds

$2\sqrt{5} \times 3\sqrt{5} = (2 \times 3) \times (\sqrt{5} \times \sqrt{5})$

Step 2: Multiply the whole numbers

$2 \times 3 = 6$

Step 3: Multiply the surds

$\sqrt{5} \times \sqrt{5} = 5$

Step 4: Multiply the two results

$6 \times 5 = 30$

ANSWER 30

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WORKED EXAMPLE 3.4

2 marks

$4\sqrt{3} \times \sqrt{6}$

Step 1: Group the whole numbers and the surds

$4\sqrt{3} \times \sqrt{6} = (4 \times 1) \times (\sqrt{3} \times \sqrt{6})$

Step 2: Multiply the whole numbers

$4 \times 1 = 4$

Step 3: Multiply the surds

$\sqrt{3} \times \sqrt{6} = \sqrt{3 \times 6} = \sqrt{18}$

Step 4: Put the results together

$4 \times \sqrt{18} = 4\sqrt{18}$

Step 5: Simplify the surd

$18 = 9 \times 2$ (9 is the largest square factor)

$4\sqrt{18} = 4 \times \sqrt{9} \times \sqrt{2} = 4 \times 3 \times \sqrt{2} = 12\sqrt{2}$

ANSWER $12\sqrt{2}$

WORKED EXAMPLE 3.5

2 marks

$(-2\sqrt{6}) \times 3\sqrt{2}$

Step 1: Group the whole numbers and the surds

$(-2\sqrt{6}) \times 3\sqrt{2} = ((-2) \times 3) \times (\sqrt{6} \times \sqrt{2})$

Step 2: Multiply the whole numbers

$(-2) \times 3 = -6$

Step 3: Multiply the surds

$\sqrt{6} \times \sqrt{2} = \sqrt{6 \times 2} = \sqrt{12}$

Step 4: Put the results together

$-6 \times \sqrt{12} = -6\sqrt{12}$

Step 5: Simplify the surd

$12 = 4 \times 3$ (4 is the largest square factor)

$-6\sqrt{12} = -6 \times \sqrt{4} \times \sqrt{3} = -6 \times 2 \times \sqrt{3} = -12\sqrt{3}$

ANSWER $-12\sqrt{3}$

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WORKED EXAMPLE 3.6

2 marks

$5\sqrt{2} \times 2\sqrt{10}$

Step 1: Group the whole numbers and the surds

$5\sqrt{2} \times 2\sqrt{10} = (5 \times 2) \times (\sqrt{2} \times \sqrt{10})$

Step 2: Multiply the whole numbers

$5 \times 2 = 10$

Step 3: Multiply the surds

$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$

Step 4: Put the results together

$10 \times \sqrt{20} = 10\sqrt{20}$

Step 5: Simplify the surd

$20 = 4 \times 5$ (4 is the largest square factor)

$10\sqrt{20} = 10 \times \sqrt{4} \times \sqrt{5} = 10 \times 2 \times \sqrt{5} = 20\sqrt{5}$

ANSWER $20\sqrt{5}$

M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. $3\sqrt{2} \times 2\sqrt{11}$ [2]
2. $4\sqrt{7} \times 2\sqrt{7}$ [2]
3. $2\sqrt{3} \times 5\sqrt{6}$ [2]
4. $(-3\sqrt{5}) \times 2\sqrt{10}$ [2]
5. $6\sqrt{2} \times \sqrt{14}$ [2]
6. $3\sqrt{6} \times 2\sqrt{15}$ [2]

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Concept 4 || Squaring and cubing surds

S || SOLVE

A power tells you how many times to multiply. Write the power out as a multiplication, then group the whole numbers and the surds. Remember that the power applies to everything inside the bracket.

KEY RULE

$(\sqrt{a})^2 = a \qquad (p\sqrt{a})^2 = p^2a \qquad (\sqrt{a})^3 = a\sqrt{a}$

WATCH OUT

× $(3\sqrt{2})^2 = 3 \times 2 = 6$

Both parts are squared: $3^2 = 9$ and $(\sqrt{2})^2 = 2$, so the answer is $9 \times 2 = 18$.

Worked examples

WORKED EXAMPLE 4.1 — 2 marks

$(\sqrt{7})^2$

Step 1: Write the square as a multiplication

$(\sqrt{7})^2 = \sqrt{7} \times \sqrt{7}$

Step 2: Multiply the surd by itself

$\sqrt{7} \times \sqrt{7} = 7$

ANSWER 7

WORKED EXAMPLE 4.2

2 marks

$(3\sqrt{2})^2$

Step 1: Write the square as a multiplication

$(3\sqrt{2})^2 = 3\sqrt{2} \times 3\sqrt{2}$

Step 2: Group the whole numbers and the surds

$= (3 \times 3) \times (\sqrt{2} \times \sqrt{2})$

Step 3: Multiply each group

$3 \times 3 = 9$

$\sqrt{2} \times \sqrt{2} = 2$

Step 4: Multiply the two results

$9 \times 2 = 18$

ANSWER 18

WORKED EXAMPLE 4.3

2 marks

$(2\sqrt{5})^2$

Step 1: Write the square as a multiplication

$(2\sqrt{5})^2 = 2\sqrt{5} \times 2\sqrt{5}$

Step 2: Group the whole numbers and the surds

$= (2 \times 2) \times (\sqrt{5} \times \sqrt{5})$

Step 3: Multiply each group

$2 \times 2 = 4$

$\sqrt{5} \times \sqrt{5} = 5$

Step 4: Multiply the two results

$4 \times 5 = 20$

ANSWER 20

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WORKED EXAMPLE 4.4

2 marks

$(-4\sqrt{3})^2$

Step 1: Write the square as a multiplication

$(-4\sqrt{3})^2 = -4\sqrt{3} \times -4\sqrt{3}$

Step 2: Group the whole numbers and the surds

$= ((-4) \times (-4)) \times (\sqrt{3} \times \sqrt{3})$

Step 3: Multiply each group

$(-4) \times (-4) = 16$

$\sqrt{3} \times \sqrt{3} = 3$

Step 4: Multiply the two results

$16 \times 3 = 48$

ANSWER **48**

WORKED EXAMPLE 4.5

2 marks

$(\sqrt{2})^3$

Step 1: Write the cube as a multiplication

$(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2}$

Step 2: Pair up two of the surds

$= (\sqrt{2} \times \sqrt{2}) \times \sqrt{2}$

$= 2 \times \sqrt{2}$

$= 2\sqrt{2}$

ANSWER **$2\sqrt{2}$**

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WORKED EXAMPLE 4.6

2 marks

$(2\sqrt{3})^3$

Step 1: Write the cube as a multiplication

$(2\sqrt{3})^3 = 2\sqrt{3} \times 2\sqrt{3} \times 2\sqrt{3}$

Step 2: Group the whole numbers and the surds

$= (2 \times 2 \times 2) \times (\sqrt{3} \times \sqrt{3} \times \sqrt{3})$

Step 3: Multiply the whole numbers

$2 \times 2 \times 2 = 8$

Step 4: Multiply the surds

$\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3 \times \sqrt{3} = 3\sqrt{3}$

Step 5: Multiply the two results

$8 \times 3\sqrt{3} = 24\sqrt{3}$

ANSWER $24\sqrt{3}$

M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. $(\sqrt{13})^2$ [2]
2. $(5\sqrt{2})^2$ [2]
3. $(3\sqrt{7})^2$ [2]
4. $(-2\sqrt{6})^2$ [2]
5. $(\sqrt{5})^3$ [2]
6. $(3\sqrt{2})^3$ [2]

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Concept 5 || Expanding a single bracket

S || SOLVE

To expand a bracket, multiply the term outside by every term inside. Each multiplication follows the rules you have already used, so simplify each result before writing the final answer.

KEY RULE

$$\sqrt{a}(b + c) = b\sqrt{a} + c\sqrt{a}$$

WATCH OUT

✗ $\sqrt{2}(3 + \sqrt{2}) = 3\sqrt{2} + \sqrt{2}$

The outside term multiplies both terms. $\sqrt{2} \times \sqrt{2} = 2$, so the answer is $3\sqrt{2} + 2$.

Worked examples

WORKED EXAMPLE 5.1 — 3 marks

$\sqrt{2}(3 + \sqrt{2})$

Step 1: Multiply $\sqrt{2}$ by the first term

$\sqrt{2} \times 3 = 3\sqrt{2}$

Step 2: Multiply $\sqrt{2}$ by the second term

$\sqrt{2} \times \sqrt{2} = 2$

Step 3: Write the results together

$\sqrt{2}(3 + \sqrt{2}) = 3\sqrt{2} + 2$

$= 2 + 3\sqrt{2}$

ANSWER $2 + 3\sqrt{2}$

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WORKED EXAMPLE 5.2

3 marks

$\sqrt{3}(\sqrt{3} - 1)$

Step 1: Multiply $\sqrt{3}$ by the first term

$\sqrt{3} \times \sqrt{3} = 3$

Step 2: Multiply $\sqrt{3}$ by the second term

$\sqrt{3} \times (-1) = -\sqrt{3}$

Step 3: Write the results together

$\sqrt{3}(\sqrt{3} - 1) = 3 - \sqrt{3}$

ANSWER **$3 - \sqrt{3}$**

WORKED EXAMPLE 5.3

3 marks

$2\sqrt{5}(\sqrt{5} + 4)$

Step 1: Multiply $2\sqrt{5}$ by the first term

$2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10$

Step 2: Multiply $2\sqrt{5}$ by the second term

$2\sqrt{5} \times 4 = 8\sqrt{5}$

Step 3: Write the results together

$2\sqrt{5}(\sqrt{5} + 4) = 10 + 8\sqrt{5}$

ANSWER **$10 + 8\sqrt{5}$**

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WORKED EXAMPLE 5.4

3 marks

$\sqrt{2}(\sqrt{6} + \sqrt{10})$

Step 1: Multiply $\sqrt{2}$ by the first term

$\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6} = \sqrt{12}$

$12 = 4 \times 3$ (4 is the largest square factor)

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Step 2: Multiply $\sqrt{2}$ by the second term

$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$

$20 = 4 \times 5$ (4 is the largest square factor)

$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

Step 3: Write the results together

$\sqrt{2}(\sqrt{6} + \sqrt{10}) = 2\sqrt{3} + 2\sqrt{5}$

ANSWER **$2\sqrt{3} + 2\sqrt{5}$**

WORKED EXAMPLE 5.5

3 marks

$3\sqrt{2}(2\sqrt{2} - \sqrt{3})$

Step 1: Multiply $3\sqrt{2}$ by the first term

$3\sqrt{2} \times 2\sqrt{2} = (3 \times 2) \times (\sqrt{2} \times \sqrt{2}) = 6 \times 2 = 12$

Step 2: Multiply $3\sqrt{2}$ by the second term

$3\sqrt{2} \times (-\sqrt{3}) = (3 \times (-1)) \times \sqrt{2 \times 3} = -3\sqrt{6}$

Step 3: Write the results together

$3\sqrt{2}(2\sqrt{2} - \sqrt{3}) = 12 - 3\sqrt{6}$

ANSWER **$12 - 3\sqrt{6}$**

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WORKED EXAMPLE 5.6

3 marks

$\sqrt{3}(4 + 2\sqrt{6})$

Step 1: Multiply $\sqrt{3}$ by the first term

$\sqrt{3} \times 4 = 4\sqrt{3}$

Step 2: Multiply $\sqrt{3}$ by the second term

$\sqrt{3} \times 2\sqrt{6} = 2 \times \sqrt{3 \times 6} = 2\sqrt{18}$

$18 = 9 \times 2$ (9 is the largest square factor)

$2\sqrt{18} = 2 \times \sqrt{9} \times \sqrt{2} = 2 \times 3 \times \sqrt{2} = 6\sqrt{2}$

Step 3: Write the results together

$\sqrt{3}(4 + 2\sqrt{6}) = 4\sqrt{3} + 6\sqrt{2}$

$= 6\sqrt{2} + 4\sqrt{3}$

ANSWER **$6\sqrt{2} + 4\sqrt{3}$**

M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. $\sqrt{5}(2 + \sqrt{5})$ [3]
2. $\sqrt{7}(\sqrt{7} - 3)$ [3]
3. $2\sqrt{3}(\sqrt{3} + 5)$ [3]
4. $\sqrt{2}(\sqrt{8} + \sqrt{6})$ [3]
5. $4\sqrt{5}(\sqrt{5} - 2\sqrt{2})$ [3]
6. $\sqrt{6}(\sqrt{2} + \sqrt{3})$ [3]

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Concept 6 || Expanding double brackets

S || SOLVE

Every term in the first bracket multiplies every term in the second bracket, which gives four products. A grid keeps them organised. Finish by collecting the whole numbers together and the like surds together.

KEY RULE

$$(a + b)(c + d) = ac + ad + bc + bd$$

WATCH OUT

× $(\sqrt{5} + 1)^2 = 5 + 1$

Squaring a bracket means multiplying it by itself. Write it as two brackets and expand fully: the middle terms $2\sqrt{5}$ must not be lost.

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Worked examples

WORKED EXAMPLE 6.1

3 marks

$(1 + \sqrt{2})(3 + \sqrt{2})$

Step 1: Multiply every term in the first bracket by every term in the second

$1 \times 3 = 3$

$1 \times \sqrt{2} = \sqrt{2}$

$\sqrt{2} \times 3 = 3\sqrt{2}$

$\sqrt{2} \times \sqrt{2} = 2$

Step 2: Fill in the grid

×	3	$\sqrt{2}$
1	3	$\sqrt{2}$
$\sqrt{2}$	$3\sqrt{2}$	2

Step 3: Collect like terms

$= 3 + \sqrt{2} + 3\sqrt{2} + 2$

Whole numbers: $3 + 2 = 5$

Surds in $\sqrt{2}$: $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$= 5 + 4\sqrt{2}$

ANSWER $5 + 4\sqrt{2}$

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WORKED EXAMPLE 6.2

3 marks

$(2 + \sqrt{3})(2 - \sqrt{3})$

Step 1: Multiply every term in the first bracket by every term in the second

$2 \times 2 = 4$

$2 \times (-\sqrt{3}) = -2\sqrt{3}$

$\sqrt{3} \times 2 = 2\sqrt{3}$

$\sqrt{3} \times (-\sqrt{3}) = (-1) \times 3 = -3$

Step 2: Fill in the grid

×	2	$-\sqrt{3}$
2	4	$-2\sqrt{3}$
$\sqrt{3}$	$2\sqrt{3}$	-3

Step 3: Collect like terms

$= 4 - 2\sqrt{3} + 2\sqrt{3} - 3$

Whole numbers: $4 + (-3) = 1$

Surds in $\sqrt{3}$: $(-2\sqrt{3}) + 2\sqrt{3} = 0$

$= 1$

ANSWER **1**

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WORKED EXAMPLE 6.3

3 marks

$(\sqrt{5} + 1)^2$

Step 1: Write the square as two brackets

$(\sqrt{5} + 1)^2 = (\sqrt{5} + 1)(\sqrt{5} + 1)$

Step 2: Multiply every term in the first bracket by every term in the second

$\sqrt{5} \times \sqrt{5} = 5$

$\sqrt{5} \times 1 = \sqrt{5}$

$1 \times \sqrt{5} = \sqrt{5}$

$1 \times 1 = 1$

Step 3: Fill in the grid

×	$\sqrt{5}$	1
$\sqrt{5}$	5	$\sqrt{5}$
1	$\sqrt{5}$	1

Step 4: Collect like terms

$= 5 + \sqrt{5} + \sqrt{5} + 1$

Whole numbers: $5 + 1 = 6$

Surds in $\sqrt{5}$: $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$

$= 6 + 2\sqrt{5}$

ANSWER $6 + 2\sqrt{5}$

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WORKED EXAMPLE 6.4

3 marks

$(3 - \sqrt{2})(1 + \sqrt{2})$

Step 1: Multiply every term in the first bracket by every term in the second

$3 \times 1 = 3$

$3 \times \sqrt{2} = 3\sqrt{2}$

$(-\sqrt{2}) \times 1 = -\sqrt{2}$

$(-\sqrt{2}) \times \sqrt{2} = (-1) \times 2 = -2$

Step 2: Fill in the grid

×	1	$\sqrt{2}$
3	3	$3\sqrt{2}$
$-\sqrt{2}$	$-\sqrt{2}$	-2

Step 3: Collect like terms

$= 3 + 3\sqrt{2} - \sqrt{2} - 2$

Whole numbers: $3 + (-2) = 1$

Surds in $\sqrt{2}$: $3\sqrt{2} + (-\sqrt{2}) = 2\sqrt{2}$

$= 1 + 2\sqrt{2}$

ANSWER **$1 + 2\sqrt{2}$**

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WORKED EXAMPLE 6.5

3 marks

$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$

Step 1: Multiply every term in the first bracket by every term in the second

$\sqrt{3} \times \sqrt{3} = 3$

$\sqrt{3} \times (-\sqrt{2}) = -1 \times \sqrt{3 \times 2} = -\sqrt{6}$

$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

$\sqrt{2} \times (-\sqrt{2}) = (-1) \times 2 = -2$

Step 2: Fill in the grid

×	$\sqrt{3}$	$-\sqrt{2}$
$\sqrt{3}$	3	$-\sqrt{6}$
$\sqrt{2}$	$\sqrt{6}$	-2

Step 3: Collect like terms

$= 3 - \sqrt{6} + \sqrt{6} - 2$

Whole numbers: $3 + (-2) = 1$

Surds in $\sqrt{6}$: $(-\sqrt{6}) + \sqrt{6} = 0$

$= 1$

ANSWER **1**

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WORKED EXAMPLE 6.6

3 marks

$(2\sqrt{5} - 1)(\sqrt{5} + 3)$

Step 1: Multiply every term in the first bracket by every term in the second

$2\sqrt{5} \times \sqrt{5} = 2 \times 5 = 10$

$2\sqrt{5} \times 3 = 6\sqrt{5}$

$(-1) \times \sqrt{5} = -\sqrt{5}$

$(-1) \times 3 = -3$

Step 2: Fill in the grid

×	$\sqrt{5}$	3
$2\sqrt{5}$	10	$6\sqrt{5}$
−1	$-\sqrt{5}$	−3

Step 3: Collect like terms

$= 10 + 6\sqrt{5} - \sqrt{5} - 3$

Whole numbers: $10 + (-3) = 7$

Surds in $\sqrt{5}$: $6\sqrt{5} + (-\sqrt{5}) = 5\sqrt{5}$

$= 7 + 5\sqrt{5}$

ANSWER **$7 + 5\sqrt{5}$**

M || MASTER: YOUR TURN

Simplify your answers fully. Answers are in the Answer Key.

1. $(2 + \sqrt{5})(1 + \sqrt{5})$ [3]
2. $(4 + \sqrt{7})(4 - \sqrt{7})$ [3]
3. $(\sqrt{3} - 2)^2$ [3]
4. $(1 + 2\sqrt{3})(2 + \sqrt{3})$ [3]
5. $(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})$ [3]
6. $(3\sqrt{2} + 1)(\sqrt{2} - 4)$ [3]

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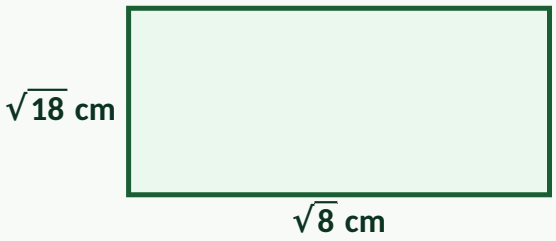
Outperform

O || OUTPERFORM

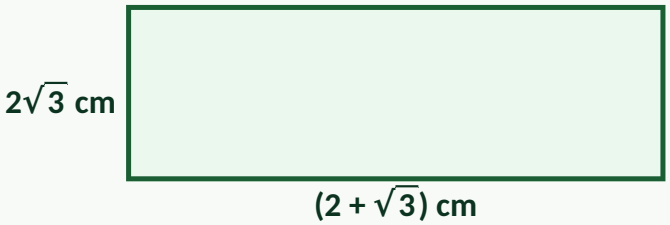
Put everything together. These questions mix the skills from all six concepts and are set out like exam questions.

CHALLENGE QUESTIONS

1. A rectangle measures $\sqrt{8}$ cm by $\sqrt{18}$ cm. Work out its area. Give your answer as a whole number. [3]



2. A square has sides of length $3\sqrt{5}$ cm. Work out its area. [2]
3. Show that $\sqrt{6} \times \sqrt{15} = 3\sqrt{10}$ [3]
4. Show that $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 2$ [3]
5. Find the value of k when $\sqrt{k} \times \sqrt{3} = 6$ [3]
6. A rectangle has width $2\sqrt{3}$ cm and length $(2 + \sqrt{3})$ cm. Show that its area is $(6 + 4\sqrt{3})$ cm^2. [4]



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Answer Key

ANSWER SUMMARY

Q	Question	Answer
CONCEPT 1 || MULTIPLYING TWO SURDS		
1.1	$\sqrt{3} \times \sqrt{5}$	$\sqrt{15}$
1.2	$\sqrt{7} \times \sqrt{2}$	$\sqrt{14}$
1.3	$\sqrt{11} \times \sqrt{11}$	11
1.4	$\sqrt{2} \times \sqrt{18}$	6
1.5	$\sqrt{5} \times \sqrt{20}$	10
1.6	$\sqrt{2} \times \sqrt{3} \times \sqrt{7}$	$\sqrt{42}$
CONCEPT 2 || SIMPLIFYING THE ANSWER		
2.1	Simplify $\sqrt{18}$	$3\sqrt{2}$
2.2	Simplify $\sqrt{75}$	$5\sqrt{3}$
2.3	$\sqrt{2} \times \sqrt{10}$	$2\sqrt{5}$
2.4	$\sqrt{3} \times \sqrt{6}$	$3\sqrt{2}$
2.5	$\sqrt{5} \times \sqrt{15}$	$5\sqrt{3}$
2.6	$\sqrt{10} \times \sqrt{8}$	$4\sqrt{5}$
CONCEPT 3 || SURDS WITH WHOLE NUMBER COEFFICIENTS		
3.1	$3\sqrt{2} \times 2\sqrt{11}$	$6\sqrt{22}$
3.2	$4\sqrt{7} \times 2\sqrt{7}$	56
3.3	$2\sqrt{3} \times 5\sqrt{6}$	$30\sqrt{2}$
3.4	$(-3\sqrt{5}) \times 2\sqrt{10}$	$-30\sqrt{2}$
3.5	$6\sqrt{2} \times \sqrt{14}$	$12\sqrt{7}$
3.6	$3\sqrt{6} \times 2\sqrt{15}$	$18\sqrt{10}$
CONCEPT 4 || SQUARING AND CUBING SURDS		
4.1	$(\sqrt{13})^2$	13
4.2	$(5\sqrt{2})^2$	50

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4.3	$(3\sqrt{7})^2$	**63**
4.4	$(-2\sqrt{6})^2$	**24**
4.5	$(\sqrt{5})^3$	**$5\sqrt{5}$**
4.6	$(3\sqrt{2})^3$	**$54\sqrt{2}$**
CONCEPT 5 || EXPANDING A SINGLE BRACKET		
5.1	$\sqrt{5}(2+\sqrt{5})$	**$5+2\sqrt{5}$**
5.2	$\sqrt{7}(\sqrt{7}-3)$	**$7-3\sqrt{7}$**
5.3	$2\sqrt{3}(\sqrt{3}+5)$	**$6+10\sqrt{3}$**
5.4	$\sqrt{2}(\sqrt{8}+\sqrt{6})$	**$4+2\sqrt{3}$**
5.5	$4\sqrt{5}(\sqrt{5}-2\sqrt{2})$	**$20-8\sqrt{10}$**
5.6	$\sqrt{6}(\sqrt{2}+\sqrt{3})$	**$3\sqrt{2}+2\sqrt{3}$**
CONCEPT 6 || EXPANDING DOUBLE BRACKETS		
6.1	$(2+\sqrt{5})(1+\sqrt{5})$	**$7+3\sqrt{5}$**
6.2	$(4+\sqrt{7})(4-\sqrt{7})$	**9**
6.3	$(\sqrt{3}-2)^2$	**$7-4\sqrt{3}$**
6.4	$(1+2\sqrt{3})(2+\sqrt{3})$	**$8+5\sqrt{3}$**
6.5	$(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$	**4**
6.6	$(3\sqrt{2}+1)(\sqrt{2}-4)$	**$2-11\sqrt{2}$**
OUTPERFORM		
C1	A rectangle measures $\sqrt{8}$ cm by $\sqrt{18}$ cm. Work out its area. Give your answer as a whole number.	**12 cm²**
C2	A square has sides of length $3\sqrt{5}$ cm. Work out its area.	**45 cm²**
C3	Show that $\sqrt{6} \times \sqrt{15} = 3\sqrt{10}$	**$3\sqrt{10}$**
C4	Show that $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = 2$	**2**
C5	Find the value of k when $\sqrt{k} \times \sqrt{3} = 6$	**k = 12**
C6	A rectangle has width $2\sqrt{3}$ cm and length $(2+\sqrt{3})$ cm. Show that its area is $(6+4\sqrt{3})$ cm².	**$6+4\sqrt{3}$ cm²**

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Full worked solutions

Concept 1 || Multiplying two surds

QUESTION 1.1 — 2 marks

$\sqrt{3} \times \sqrt{5}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5}$

$= \sqrt{15}$

Step 2: Check for square factors

15 has no square factor other than 1, so $\sqrt{15}$ is already in simplest form

ANSWER $\sqrt{15}$

QUESTION 1.2 — 2 marks

$\sqrt{7} \times \sqrt{2}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{7} \times \sqrt{2} = \sqrt{7 \times 2}$

$= \sqrt{14}$

Step 2: Check for square factors

14 has no square factor other than 1, so $\sqrt{14}$ is already in simplest form

ANSWER $\sqrt{14}$

QUESTION 1.3 — 2 marks

$\sqrt{11} \times \sqrt{11}$

Step 1: Multiply the surd by itself

$\sqrt{11} \times \sqrt{11} = \sqrt{11 \times 11}$

$= \sqrt{121}$

= 11 (a surd multiplied by itself gives the number under the root)

ANSWER 11

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QUESTION 1.4

2 marks

$\sqrt{2} \times \sqrt{18}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{18} = \sqrt{2 \times 18}$

$= \sqrt{36}$

Step 2: Simplify the surd

36 is a square number, so $\sqrt{36} = 6$

ANSWER **6**

QUESTION 1.5

2 marks

$\sqrt{5} \times \sqrt{20}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{5} \times \sqrt{20} = \sqrt{5 \times 20}$

$= \sqrt{100}$

Step 2: Simplify the surd

100 is a square number, so $\sqrt{100} = 10$

ANSWER **10**

QUESTION 1.6

2 marks

$\sqrt{2} \times \sqrt{3} \times \sqrt{7}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{3} \times \sqrt{7} = \sqrt{2 \times 3 \times 7}$

$= \sqrt{42}$

Step 2: Check for square factors

42 has no square factor other than 1, so $\sqrt{42}$ is already in simplest form

ANSWER **$\sqrt{42}$**

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Concept 2 || Simplifying the answer

QUESTION 2.1

2 marks

Simplify $\sqrt{18}$

Step 1: List the square numbers

4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Step 2: Find the largest square factor of 18

$18 = 9 \times 2$

Step 3: Split the root into two roots

$\sqrt{18} = \sqrt{9} \times \sqrt{2}$

Step 4: Square root the square number

$= 3 \times \sqrt{2}$

$= 3\sqrt{2}$

ANSWER $3\sqrt{2}$

QUESTION 2.2

2 marks

Simplify $\sqrt{75}$

Step 1: List the square numbers

4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Step 2: Find the largest square factor of 75

$75 = 25 \times 3$

Step 3: Split the root into two roots

$\sqrt{75} = \sqrt{25} \times \sqrt{3}$

Step 4: Square root the square number

$= 5 \times \sqrt{3}$

$= 5\sqrt{3}$

ANSWER $5\sqrt{3}$

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QUESTION 2.3

2 marks

$\sqrt{2} \times \sqrt{10}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10}$

$= \sqrt{20}$

Step 2: Simplify the surd

$20 = 4 \times 5$ (4 is the largest square factor)

$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

ANSWER $2\sqrt{5}$

QUESTION 2.4

2 marks

$\sqrt{3} \times \sqrt{6}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{6} = \sqrt{3 \times 6}$

$= \sqrt{18}$

Step 2: Simplify the surd

$18 = 9 \times 2$ (9 is the largest square factor)

$\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

ANSWER $3\sqrt{2}$

QUESTION 2.5

2 marks

$\sqrt{5} \times \sqrt{15}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15}$

$= \sqrt{75}$

Step 2: Simplify the surd

$75 = 25 \times 3$ (25 is the largest square factor)

$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

ANSWER $5\sqrt{3}$

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QUESTION 2.6

2 marks

$\sqrt{10} \times \sqrt{8}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{10} \times \sqrt{8} = \sqrt{10 \times 8}$

$= \sqrt{80}$

Step 2: Simplify the surd

$80 = 16 \times 5$ (16 is the largest square factor)

$\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

ANSWER $4\sqrt{5}$

Concept 3 || Surds with whole number coefficients

QUESTION 3.1

2 marks

$3\sqrt{2} \times 2\sqrt{11}$

Step 1: Group the whole numbers and the surds

$3\sqrt{2} \times 2\sqrt{11} = (3 \times 2) \times (\sqrt{2} \times \sqrt{11})$

Step 2: Multiply the whole numbers

$3 \times 2 = 6$

Step 3: Multiply the surds

$\sqrt{2} \times \sqrt{11} = \sqrt{2 \times 11} = \sqrt{22}$

Step 4: Put the results together

$6 \times \sqrt{22} = 6\sqrt{22}$

ANSWER $6\sqrt{22}$

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QUESTION 3.2

2 marks

$4\sqrt{7} \times 2\sqrt{7}$

Step 1: Group the whole numbers and the surds

$4\sqrt{7} \times 2\sqrt{7} = (4 \times 2) \times (\sqrt{7} \times \sqrt{7})$

Step 2: Multiply the whole numbers

$4 \times 2 = 8$

Step 3: Multiply the surds

$\sqrt{7} \times \sqrt{7} = 7$

Step 4: Multiply the two results

$8 \times 7 = 56$

ANSWER **56**

QUESTION 3.3

2 marks

$2\sqrt{3} \times 5\sqrt{6}$

Step 1: Group the whole numbers and the surds

$2\sqrt{3} \times 5\sqrt{6} = (2 \times 5) \times (\sqrt{3} \times \sqrt{6})$

Step 2: Multiply the whole numbers

$2 \times 5 = 10$

Step 3: Multiply the surds

$\sqrt{3} \times \sqrt{6} = \sqrt{3 \times 6} = \sqrt{18}$

Step 4: Put the results together

$10 \times \sqrt{18} = 10\sqrt{18}$

Step 5: Simplify the surd

$18 = 9 \times 2$ (9 is the largest square factor)

$10\sqrt{18} = 10 \times \sqrt{9} \times \sqrt{2} = 10 \times 3 \times \sqrt{2} = 30\sqrt{2}$

ANSWER **$30\sqrt{2}$**

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QUESTION 3.4

2 marks

$(-3\sqrt{5}) \times 2\sqrt{10}$

Step 1: Group the whole numbers and the surds

$(-3\sqrt{5}) \times 2\sqrt{10} = ((-3) \times 2) \times (\sqrt{5} \times \sqrt{10})$

Step 2: Multiply the whole numbers

$(-3) \times 2 = -6$

Step 3: Multiply the surds

$\sqrt{5} \times \sqrt{10} = \sqrt{5 \times 10} = \sqrt{50}$

Step 4: Put the results together

$-6 \times \sqrt{50} = -6\sqrt{50}$

Step 5: Simplify the surd

$50 = 25 \times 2$ (25 is the largest square factor)

$-6\sqrt{50} = -6 \times \sqrt{25} \times \sqrt{2} = -6 \times 5 \times \sqrt{2} = -30\sqrt{2}$

ANSWER $-30\sqrt{2}$

QUESTION 3.5

2 marks

$6\sqrt{2} \times \sqrt{14}$

Step 1: Group the whole numbers and the surds

$6\sqrt{2} \times \sqrt{14} = (6 \times 1) \times (\sqrt{2} \times \sqrt{14})$

Step 2: Multiply the whole numbers

$6 \times 1 = 6$

Step 3: Multiply the surds

$\sqrt{2} \times \sqrt{14} = \sqrt{2 \times 14} = \sqrt{28}$

Step 4: Put the results together

$6 \times \sqrt{28} = 6\sqrt{28}$

Step 5: Simplify the surd

$28 = 4 \times 7$ (4 is the largest square factor)

$6\sqrt{28} = 6 \times \sqrt{4} \times \sqrt{7} = 6 \times 2 \times \sqrt{7} = 12\sqrt{7}$

ANSWER $12\sqrt{7}$

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QUESTION 3.6

2 marks

$3\sqrt{6} \times 2\sqrt{15}$

Step 1: Group the whole numbers and the surds

$3\sqrt{6} \times 2\sqrt{15} = (3 \times 2) \times (\sqrt{6} \times \sqrt{15})$

Step 2: Multiply the whole numbers

$3 \times 2 = 6$

Step 3: Multiply the surds

$\sqrt{6} \times \sqrt{15} = \sqrt{6 \times 15} = \sqrt{90}$

Step 4: Put the results together

$6 \times \sqrt{90} = 6\sqrt{90}$

Step 5: Simplify the surd

$90 = 9 \times 10$ (9 is the largest square factor)

$6\sqrt{90} = 6 \times \sqrt{9} \times \sqrt{10} = 6 \times 3 \times \sqrt{10} = 18\sqrt{10}$

ANSWER **$18\sqrt{10}$**

Concept 4 || Squaring and cubing surds

QUESTION 4.1

2 marks

$(\sqrt{13})^2$

Step 1: Write the square as a multiplication

$(\sqrt{13})^2 = \sqrt{13} \times \sqrt{13}$

Step 2: Multiply the surd by itself

$\sqrt{13} \times \sqrt{13} = 13$

ANSWER **13**

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QUESTION 4.2

2 marks

$(5\sqrt{2})^2$

Step 1: Write the square as a multiplication

$(5\sqrt{2})^2 = 5\sqrt{2} \times 5\sqrt{2}$

Step 2: Group the whole numbers and the surds

$= (5 \times 5) \times (\sqrt{2} \times \sqrt{2})$

Step 3: Multiply each group

$5 \times 5 = 25$

$\sqrt{2} \times \sqrt{2} = 2$

Step 4: Multiply the two results

$25 \times 2 = 50$

ANSWER 50

QUESTION 4.3

2 marks

$(3\sqrt{7})^2$

Step 1: Write the square as a multiplication

$(3\sqrt{7})^2 = 3\sqrt{7} \times 3\sqrt{7}$

Step 2: Group the whole numbers and the surds

$= (3 \times 3) \times (\sqrt{7} \times \sqrt{7})$

Step 3: Multiply each group

$3 \times 3 = 9$

$\sqrt{7} \times \sqrt{7} = 7$

Step 4: Multiply the two results

$9 \times 7 = 63$

ANSWER 63

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QUESTION 4.4

2 marks

$(-2\sqrt{6})^2$

Step 1: Write the square as a multiplication

$(-2\sqrt{6})^2 = -2\sqrt{6} \times -2\sqrt{6}$

Step 2: Group the whole numbers and the surds

$= ((-2) \times (-2)) \times (\sqrt{6} \times \sqrt{6})$

Step 3: Multiply each group

$(-2) \times (-2) = 4$

$\sqrt{6} \times \sqrt{6} = 6$

Step 4: Multiply the two results

$4 \times 6 = 24$

ANSWER 24

QUESTION 4.5

2 marks

$(\sqrt{5})^3$

Step 1: Write the cube as a multiplication

$(\sqrt{5})^3 = \sqrt{5} \times \sqrt{5} \times \sqrt{5}$

Step 2: Pair up two of the surds

$= (\sqrt{5} \times \sqrt{5}) \times \sqrt{5}$

$= 5 \times \sqrt{5}$

$= 5\sqrt{5}$

ANSWER $5\sqrt{5}$

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QUESTION 4.6

2 marks

$(3\sqrt{2})^3$

Step 1: Write the cube as a multiplication

$(3\sqrt{2})^3 = 3\sqrt{2} \times 3\sqrt{2} \times 3\sqrt{2}$

Step 2: Group the whole numbers and the surds

$= (3 \times 3 \times 3) \times (\sqrt{2} \times \sqrt{2} \times \sqrt{2})$

Step 3: Multiply the whole numbers

$3 \times 3 \times 3 = 27$

Step 4: Multiply the surds

$\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$

Step 5: Multiply the two results

$27 \times 2\sqrt{2} = 54\sqrt{2}$

ANSWER $54\sqrt{2}$

Concept 5 || Expanding a single bracket

QUESTION 5.1

3 marks

$\sqrt{5}(2 + \sqrt{5})$

Step 1: Multiply $\sqrt{5}$ by the first term

$\sqrt{5} \times 2 = 2\sqrt{5}$

Step 2: Multiply $\sqrt{5}$ by the second term

$\sqrt{5} \times \sqrt{5} = 5$

Step 3: Write the results together

$\sqrt{5}(2 + \sqrt{5}) = 2\sqrt{5} + 5$

$= 5 + 2\sqrt{5}$

ANSWER $5 + 2\sqrt{5}$

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QUESTION 5.2

3 marks

$\sqrt{7}(\sqrt{7} - 3)$

Step 1: Multiply $\sqrt{7}$ by the first term

$\sqrt{7} \times \sqrt{7} = 7$

Step 2: Multiply $\sqrt{7}$ by the second term

$\sqrt{7} \times (-3) = -3\sqrt{7}$

Step 3: Write the results together

$\sqrt{7}(\sqrt{7} - 3) = 7 - 3\sqrt{7}$

ANSWER **$7 - 3\sqrt{7}$**

QUESTION 5.3

3 marks

$2\sqrt{3}(\sqrt{3} + 5)$

Step 1: Multiply $2\sqrt{3}$ by the first term

$2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$

Step 2: Multiply $2\sqrt{3}$ by the second term

$2\sqrt{3} \times 5 = 10\sqrt{3}$

Step 3: Write the results together

$2\sqrt{3}(\sqrt{3} + 5) = 6 + 10\sqrt{3}$

ANSWER **$6 + 10\sqrt{3}$**

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QUESTION 5.4

3 marks

$\sqrt{2}(\sqrt{8}+\sqrt{6})$

Step 1: Multiply $\sqrt{2}$ by the first term

$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16}$

16 is a square number, so $\sqrt{16} = 4$

Step 2: Multiply $\sqrt{2}$ by the second term

$\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6} = \sqrt{12}$

$12 = 4 \times 3$ (4 is the largest square factor)

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Step 3: Write the results together

$\sqrt{2}(\sqrt{8}+\sqrt{6}) = 4 + 2\sqrt{3}$

ANSWER **$4 + 2\sqrt{3}$**

QUESTION 5.5

3 marks

$4\sqrt{5}(\sqrt{5}-2\sqrt{2})$

Step 1: Multiply $4\sqrt{5}$ by the first term

$4\sqrt{5} \times \sqrt{5} = 4 \times 5 = 20$

Step 2: Multiply $4\sqrt{5}$ by the second term

$4\sqrt{5} \times (-2\sqrt{2}) = (4 \times (-2)) \times \sqrt{5 \times 2} = -8\sqrt{10}$

Step 3: Write the results together

$4\sqrt{5}(\sqrt{5}-2\sqrt{2}) = 20 - 8\sqrt{10}$

ANSWER **$20 - 8\sqrt{10}$**

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QUESTION 5.6

3 marks

$\sqrt{6}(\sqrt{2}+\sqrt{3})$

Step 1: Multiply $\sqrt{6}$ by the first term

$\sqrt{6} \times \sqrt{2} = \sqrt{6 \times 2} = \sqrt{12}$

$12 = 4 \times 3$ (4 is the largest square factor)

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Step 2: Multiply $\sqrt{6}$ by the second term

$\sqrt{6} \times \sqrt{3} = \sqrt{6 \times 3} = \sqrt{18}$

$18 = 9 \times 2$ (9 is the largest square factor)

$\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

Step 3: Write the results together

$\sqrt{6}(\sqrt{2}+\sqrt{3}) = 2\sqrt{3} + 3\sqrt{2}$

$= 3\sqrt{2} + 2\sqrt{3}$

ANSWER $3\sqrt{2} + 2\sqrt{3}$

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Concept 6 || Expanding double brackets

QUESTION 6.1

3 marks

$(2 + \sqrt{5})(1 + \sqrt{5})$

Step 1: Multiply every term in the first bracket by every term in the second

$2 \times 1 = 2$

$2 \times \sqrt{5} = 2\sqrt{5}$

$\sqrt{5} \times 1 = \sqrt{5}$

$\sqrt{5} \times \sqrt{5} = 5$

Step 2: Fill in the grid

×	1	$\sqrt{5}$
2	2	$2\sqrt{5}$
$\sqrt{5}$	$\sqrt{5}$	5

Step 3: Collect like terms

$= 2 + 2\sqrt{5} + \sqrt{5} + 5$

Whole numbers: $2 + 5 = 7$

Surds in $\sqrt{5}$: $2\sqrt{5} + \sqrt{5} = 3\sqrt{5}$

$= 7 + 3\sqrt{5}$

ANSWER $7 + 3\sqrt{5}$

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QUESTION 6.2

3 marks

$(4 + \sqrt{7})(4 - \sqrt{7})$

Step 1: Multiply every term in the first bracket by every term in the second

$4 \times 4 = 16$

$4 \times (-\sqrt{7}) = -4\sqrt{7}$

$\sqrt{7} \times 4 = 4\sqrt{7}$

$\sqrt{7} \times (-\sqrt{7}) = (-1) \times 7 = -7$

Step 2: Fill in the grid

×	4	$-\sqrt{7}$
4	16	$-4\sqrt{7}$
$\sqrt{7}$	$4\sqrt{7}$	-7

Step 3: Collect like terms

$= 16 - 4\sqrt{7} + 4\sqrt{7} - 7$

Whole numbers: $16 + (-7) = 9$

Surds in $\sqrt{7}$: $(-4\sqrt{7}) + 4\sqrt{7} = 0$

$= 9$

ANSWER **9**

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QUESTION 6.3

3 marks

$(\sqrt{3} - 2)^2$

Step 1: Write the square as two brackets

$(\sqrt{3} - 2)^2 = (\sqrt{3} - 2)(\sqrt{3} - 2)$

Step 2: Multiply every term in the first bracket by every term in the second

$\sqrt{3} \times \sqrt{3} = 3$

$\sqrt{3} \times (-2) = -2\sqrt{3}$

$(-2) \times \sqrt{3} = -2\sqrt{3}$

$(-2) \times (-2) = 4$

Step 3: Fill in the grid

×	$\sqrt{3}$	−2
$\sqrt{3}$	3	$-2\sqrt{3}$
−2	$-2\sqrt{3}$	4

Step 4: Collect like terms

$= 3 - 2\sqrt{3} - 2\sqrt{3} + 4$

Whole numbers: $3 + 4 = 7$

Surds in $\sqrt{3}$: $(-2\sqrt{3}) + (-2\sqrt{3}) = -4\sqrt{3}$

$= 7 - 4\sqrt{3}$

ANSWER $7 - 4\sqrt{3}$

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QUESTION 6.4

3 marks

$(1 + 2\sqrt{3})(2 + \sqrt{3})$

Step 1: Multiply every term in the first bracket by every term in the second

$1 \times 2 = 2$

$1 \times \sqrt{3} = \sqrt{3}$

$2\sqrt{3} \times 2 = 4\sqrt{3}$

$2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$

Step 2: Fill in the grid

×	**2**	**$\sqrt{3}$**
1	2	$\sqrt{3}$
$2\sqrt{3}$	$4\sqrt{3}$	6

Step 3: Collect like terms

$= 2 + \sqrt{3} + 4\sqrt{3} + 6$

Whole numbers: $2 + 6 = 8$

Surds in $\sqrt{3}$: $\sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$

$= 8 + 5\sqrt{3}$

ANSWER **$8 + 5\sqrt{3}$**

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QUESTION 6.5

3 marks

$(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$

Step 1: Multiply every term in the first bracket by every term in the second

$\sqrt{6} \times \sqrt{6} = 6$

$\sqrt{6} \times (-\sqrt{2}) = -1 \times \sqrt{6 \times 2} = -\sqrt{12}$

$12 = 4 \times 3$ (4 is the largest square factor)

$-\sqrt{12} = -1 \times \sqrt{4} \times \sqrt{3} = -1 \times 2 \times \sqrt{3} = -2\sqrt{3}$

$\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6} = \sqrt{12}$

$12 = 4 \times 3$ (4 is the largest square factor)

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$\sqrt{2} \times (-\sqrt{2}) = (-1) \times 2 = -2$

Step 2: Fill in the grid

×	$\sqrt{6}$	$-\sqrt{2}$
$\sqrt{6}$	6	$-2\sqrt{3}$
$\sqrt{2}$	$2\sqrt{3}$	-2

Step 3: Collect like terms

$= 6 - 2\sqrt{3} + 2\sqrt{3} - 2$

Whole numbers: $6 + (-2) = 4$

Surds in $\sqrt{3}$: $(-2\sqrt{3}) + 2\sqrt{3} = 0$

$= 4$

ANSWER 4

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QUESTION 6.6

3 marks

$(3\sqrt{2} + 1)(\sqrt{2} - 4)$

Step 1: Multiply every term in the first bracket by every term in the second

$3\sqrt{2} \times \sqrt{2} = 3 \times 2 = 6$

$3\sqrt{2} \times (-4) = -12\sqrt{2}$

$1 \times \sqrt{2} = \sqrt{2}$

$1 \times (-4) = -4$

Step 2: Fill in the grid

×	$\sqrt{2}$	−4
$3\sqrt{2}$	6	$-12\sqrt{2}$
1	$\sqrt{2}$	−4

Step 3: Collect like terms

$= 6 - 12\sqrt{2} + \sqrt{2} - 4$

Whole numbers: $6 + (-4) = 2$

Surds in $\sqrt{2}$: $(-12\sqrt{2}) + \sqrt{2} = -11\sqrt{2}$

$= 2 - 11\sqrt{2}$

ANSWER **$2 - 11\sqrt{2}$**

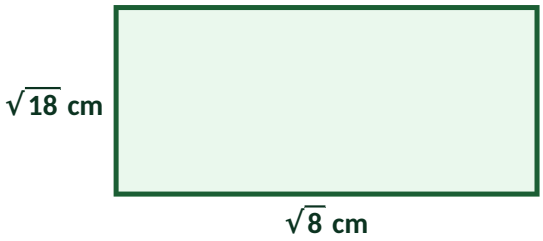
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Outperform

CHALLENGE C1

3 marks

A rectangle measures $\sqrt{8}$ cm by $\sqrt{18}$ cm. Work out its area. Give your answer as a whole number.



Step 1: Write the area as length × width

Area = $\sqrt{8} \times \sqrt{18}$

Step 2: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{8} \times \sqrt{18} = \sqrt{8 \times 18}$

$= \sqrt{144}$

Step 3: Simplify the surd

144 is a square number, so $\sqrt{144} = 12$

ANSWER	12 cm²

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CHALLENGE C2

2 marks

A square has sides of length $3\sqrt{5}$ cm. Work out its area.

Step 1: Area of a square is side × side

Area = $(3\sqrt{5})^2$

Step 2: Write the square as a multiplication

$(3\sqrt{5})^2 = 3\sqrt{5} \times 3\sqrt{5}$

Step 3: Group the whole numbers and the surds

$= (3 \times 3) \times (\sqrt{5} \times \sqrt{5})$

Step 4: Multiply each group

$3 \times 3 = 9$

$\sqrt{5} \times \sqrt{5} = 5$

Step 5: Multiply the two results

$9 \times 5 = 45$

ANSWER **45 cm²**

CHALLENGE C3

3 marks

Show that $\sqrt{6} \times \sqrt{15} = 3\sqrt{10}$

Step 1: Use the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{6} \times \sqrt{15} = \sqrt{6 \times 15}$

$= \sqrt{90}$

Step 2: Simplify the surd

$90 = 9 \times 10$ (9 is the largest square factor)

$\sqrt{90} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$

$= 3\sqrt{10}$ as required ✓

ANSWER **$3\sqrt{10}$**

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CHALLENGE C4

3 marks

Show that $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 2$

Step 1: Multiply every term in the first bracket by every term in the second

$\sqrt{5} \times \sqrt{5} = 5$

$\sqrt{5} \times (-\sqrt{3}) = -1 \times \sqrt{5 \times 3} = -\sqrt{15}$

$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$

$\sqrt{3} \times (-\sqrt{3}) = (-1) \times 3 = -3$

Step 2: Fill in the grid

×	$\sqrt{5}$	$-\sqrt{3}$
$\sqrt{5}$	5	$-\sqrt{15}$
$\sqrt{3}$	$\sqrt{15}$	-3

Step 3: Collect like terms

$= 5 - \sqrt{15} + \sqrt{15} - 3$

Whole numbers: $5 + (-3) = 2$

Surds in $\sqrt{15}$: $(-\sqrt{15}) + \sqrt{15} = 0$

$= 2$

$= 2$ as required ✓

ANSWER 2

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CHALLENGE C5

3 marks

Find the value of k when $\sqrt{k} \times \sqrt{3} = 6$

Step 1: Use the rule to combine the surds

$\sqrt{k} \times \sqrt{3} = \sqrt{3k}$

so $\sqrt{3k} = 6$

Step 2: Square both sides to remove the root

$3k = 6^2$

$3k = 36$

Step 3: Divide both sides by 3

$k = 36 \div 3$

$k = 12$

Step 4: Check

$\sqrt{12} \times \sqrt{3} = \sqrt{36} = 6$ ✓

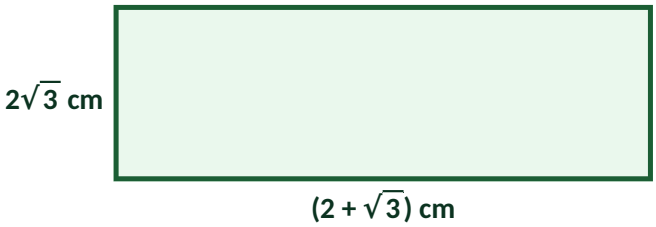
ANSWER	k = 12

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CHALLENGE C6

4 marks

A rectangle has width $2\sqrt{3}$ cm and length $(2 + \sqrt{3})$ cm. Show that its area is $(6 + 4\sqrt{3})$ cm².



Step 1: Write the area as width × length

Area = $2\sqrt{3}(2 + \sqrt{3})$

Step 2: Multiply $2\sqrt{3}$ by the first term

$2\sqrt{3} \times 2 = 4\sqrt{3}$

Step 3: Multiply $2\sqrt{3}$ by the second term

$2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$

Step 4: Write the results together

Area = $4\sqrt{3} + 6$

= $6 + 4\sqrt{3}$ cm² as required ✓

ANSWER $6 + 4\sqrt{3}$ cm²

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