Sismo Learning

3, 7, 11, 15 ... 1, 4, 9, 16 ... 1, 1, 2, 3, 5 ...

LESSON NOTE

Year 8 || Maths
Sequences

Term to term rules, the nth term and special sequences. Five concepts, 30 worked examples, 25 practice questions and a full answer key.

01 Term to term rules

02 Position to term rules

03 Finding the nth term

04 Is a number in the sequence?

05 Special sequences

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Before we start

Sequences are patterns of numbers. By the end of this lesson note you will be able to describe, continue and predict any linear sequence, and recognise the special sequences that appear again and again in maths.

Learning objectives

- ✓ Generate a sequence from a term to term rule
- ✓ Generate a sequence from a position to term rule (nth term)
- ✓ Find the nth term of an ascending or descending linear sequence
- ✓ Decide whether a number is a term in a sequence
- ✓ Recognise square, cube, triangular and Fibonacci sequences

You should already know

- ✓ Times tables and square numbers
- ✓ Adding and subtracting negative numbers
- ✓ Solving two step equations
- ✓ Substituting into an expression

Key vocabulary

sequence · term · position · term to term rule · position to term rule · nth term · linear · common difference · zeroth term · ascending · descending · square numbers · cube numbers · triangular numbers · Fibonacci

How to use this note

Each concept opens with the key idea and a method. Study the six worked examples, covering the steps and answer, then try the five practice questions. Full worked solutions are in the Answer Key at the back.

Notation

n is the position of a term: $n = 1$ for the 1st term, $n = 2$ for the 2nd term and so on. − is a minus sign.

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CONCEPT 1

Concept 1 || Term to term rules

01

Key idea

A **sequence** is a list of numbers that follow a rule. Each number in the list is called a **term**.

A **term to term rule** tells you how to get from one term to the next, for example *add 5* or *multiply by 2*.

When the same number is added or subtracted every time, the sequence is **linear** (also called arithmetic). The amount it changes by is the **common difference**.

Method

1. Find the difference: second term minus first term.
2. Check the difference is the same all the way along.
3. Keep applying the rule to extend the sequence.

Example table

Sequence	Rule	Linear?
3, 7, 11, 15	add 4	Yes
20, 17, 14, 11	subtract 3	Yes
2, 4, 8, 16	multiply by 2	No

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Worked examples

Example 1.1 [2 marks]

A sequence starts at 4. The term to term rule is **add 7**. Write down the first five terms.

Step 1: Write down the first term

1st term = 4

Step 2: Apply the rule (add 7) to each term in turn

2nd term = 4 + 7 = 11
3rd term = 11 + 7 = 18
4th term = 18 + 7 = 25
5th term = 25 + 7 = 32

ANSWER 4, 11, 18, 25, 32

Example 1.2 [2 marks]

A sequence starts at 50. The term to term rule is **subtract 6**. Write down the first five terms.

Step 1: Write down the first term

1st term = 50

Step 2: Apply the rule (subtract 6) to each term in turn

2nd term = 50 − 6 = 44
3rd term = 44 − 6 = 38
4th term = 38 − 6 = 32
5th term = 32 − 6 = 26

ANSWER 50, 44, 38, 32, 26

Example 1.3 [2 marks]

A sequence starts at 3. The term to term rule is **multiply by 2**. Write down the first five terms.

Step 1: Write down the first term

1st term = 3

Step 2: Apply the rule (multiply by 2) to each term in turn

2nd term = 3 × 2 = 6
3rd term = 6 × 2 = 12
4th term = 12 × 2 = 24
5th term = 24 × 2 = 48

ANSWER 3, 6, 12, 24, 48

Example 1.4 [3 marks]

Here is a sequence: 7, 12, 17, 22, ...
Find the term to term rule and the next two terms.

Step 1: Find the difference between consecutive terms

12 − 7 = 5
17 − 12 = 5
22 − 17 = 5

Step 2: State the term to term rule

The difference is always 5, so the rule is **add 5**.

Step 3: Apply the rule to find the next terms

22 + 5 = 27
27 + 5 = 32

ANSWER Rule: add 5. Next two terms: 27, 32

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Example 1.5 [3 marks]

Here is a sequence: 2.5, 3.1, 3.7, 4.3, ...

Find the term to term rule and the next two terms.

Step 1: Find the difference between consecutive terms

3.1 − 2.5 = 0.6

3.7 − 3.1 = 0.6

4.3 − 3.7 = 0.6

Step 2: State the term to term rule

The difference is always 0.6, so the rule is **add 0.6**.

Step 3: Apply the rule to find the next terms

4.3 + 0.6 = 4.9

4.9 + 0.6 = 5.5

ANSWER Rule: add 0.6. Next two terms: 4.9, 5.5

Example 1.6 [3 marks]

Here is a sequence: 4, −1, −6, −11, ...

Find the term to term rule and the next two terms.

Step 1: Find the difference between consecutive terms

(−1) − 4 = −5

(−6) − (−1) = −5

(−11) − (−6) = −5

Step 2: State the term to term rule

The difference is always −5, so the rule is **subtract 5**.

Step 3: Apply the rule to find the next terms

−11 − 5 = −16

−16 − 5 = −21

ANSWER Rule: subtract 5. Next two terms: −16, −21

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Practice questions

Question 1.1 [2 marks]

A sequence starts at 9. The term to term rule is **add 4**. Write down the first five terms.

Question 1.2 [2 marks]

A sequence starts at 100. The term to term rule is **subtract 15**. Write down the first five terms.

Question 1.3 [3 marks]

Here is a sequence: 13, 21, 29, 37, ...
Find the term to term rule and the next two terms.

Question 1.4 [3 marks]

Here is a sequence: 6, 2, −2, −6, ...
Find the term to term rule and the next two terms.

Question 1.5 [3 marks]

The terms of this linear sequence go up by the same amount each time. Find the missing terms.
5, ___, 17, ___, 29

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CONCEPT 2

Concept 2 || Position to term rules

02

Key idea

Every term has a **position**: 1st, 2nd, 3rd and so on. We call the position n.

A **position to term rule**, also called the **nth term**, lets you work out any term directly, without listing all the terms before it.

To find a term, **substitute** its position number for n.

Method

1. Write down the nth term rule.
2. Replace n with the position you need.
3. Multiply first, then add or subtract (order of operations).

Example table

Position n	1	2	3	4
Term ($3n + 2$)	5	8	11	14

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Worked examples

Example 2.1 [2 marks]

The nth term of a sequence is $3n + 2$. Write down the first five terms.

Step 1: Write down the nth term rule

nth term = $3n + 2$

Step 2: Substitute each position number for n

$n = 1$: $3 \times 1 + 2 = 3 + 2 = 5$
$n = 2$: $3 \times 2 + 2 = 6 + 2 = 8$
$n = 3$: $3 \times 3 + 2 = 9 + 2 = 11$
$n = 4$: $3 \times 4 + 2 = 12 + 2 = 14$
$n = 5$: $3 \times 5 + 2 = 15 + 2 = 17$

ANSWER 5, 8, 11, 14, 17

Example 2.2 [3 marks]

The nth term of a sequence is $5n - 4$. Find the first three terms and the 10th term.

Step 1: Write down the nth term rule

nth term = $5n - 4$

Step 2: Substitute each position number for n

$n = 1$: $5 \times 1 - 4 = 5 - 4 = 1$
$n = 2$: $5 \times 2 - 4 = 10 - 4 = 6$
$n = 3$: $5 \times 3 - 4 = 15 - 4 = 11$
$n = 10$: $5 \times 10 - 4 = 50 - 4 = 46$

ANSWER First three terms: 1, 6, 11. 10th term: 46

Example 2.3 [2 marks]

The nth term of a sequence is $20 - 3n$. Write down the first four terms.

Step 1: Write down the nth term rule

nth term = $20 - 3n$

Step 2: Substitute each position number for n

$n = 1$: $20 - 3 \times 1 = 20 - 3 = 17$
$n = 2$: $20 - 3 \times 2 = 20 - 6 = 14$
$n = 3$: $20 - 3 \times 3 = 20 - 9 = 11$
$n = 4$: $20 - 3 \times 4 = 20 - 12 = 8$

ANSWER 17, 14, 11, 8

Example 2.4 [2 marks]

The nth term of a sequence is $4n + 1$. Find the 50th term.

Step 1: Write down the nth term rule

nth term = $4n + 1$

Step 2: Substitute each position number for n

$n = 50$: $4 \times 50 + 1 = 200 + 1 = 201$

ANSWER 50th term = 201

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Example 2.5 [2 marks]

The nth term of a sequence is $n^2 + 3$. Write down the first four terms.

Step 1: Write down the nth term rule

nth term = $n^2 + 3$

Step 2: Substitute each position number for n

$n = 1$: $1^2 + 3 = 1 + 3 = 4$

$n = 2$: $2^2 + 3 = 4 + 3 = 7$

$n = 3$: $3^2 + 3 = 9 + 3 = 12$

$n = 4$: $4^2 + 3 = 16 + 3 = 19$

ANSWER 4, 7, 12, 19

Example 2.6 [2 marks]

The nth term of a sequence is $2n - 7$. Write down the first three terms.

Step 1: Write down the nth term rule

nth term = $2n - 7$

Step 2: Substitute each position number for n

$n = 1$: $2 \times 1 - 7 = 2 - 7 = -5$

$n = 2$: $2 \times 2 - 7 = 4 - 7 = -3$

$n = 3$: $2 \times 3 - 7 = 6 - 7 = -1$

ANSWER −5, −3, −1

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Practice questions

Question 2.1 [2 marks]

The nth term of a sequence is $2n + 5$. Write down the first five terms.

Question 2.2 [2 marks]

The nth term of a sequence is $6n - 1$. Find the 20th term.

Question 2.3 [2 marks]

The nth term of a sequence is $30 - 4n$. Write down the first four terms.

Question 2.4 [2 marks]

The nth term of a sequence is $n^2 - 1$. Write down the first four terms.

Question 2.5 [3 marks]

The nth term of a sequence is $7n - 10$. Find the first three terms and the 100th term.

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CONCEPT 3

Concept 3 || Finding the nth term

03

Key idea

For a linear sequence, the nth term always has the form **d*n* + c**.

d is the common difference. **c** is the **zeroth term**: the number that would come before the first term.

Example: 5, 8, 11, 14 goes up in 3s, so it is linked to the 3 times table (3, 6, 9, 12). Every term is 2 more, so the nth term is 3*n* + 2.

Method

1. Find the common difference d.
2. Find the zeroth term: first term minus d.
3. Write the rule as d*n* + zeroth term.
4. Check your rule using a term you already know.

Example table

Position *n*	0	1	2	3	4
3*n*	0	3	6	9	12
Term	2	5	8	11	14

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Worked examples

Example 3.1 [2 marks]

Find the nth term of the sequence 5, 8, 11, 14, ...

Step 1: Find the common difference

$8 - 5 = 3$

$11 - 8 = 3$

$14 - 11 = 3$

Common difference = 3

Step 2: Start the rule with the difference times *n*

The nth term begins $3n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 5 - 3 = 2$

Step 4: Write the nth term

nth term = $3n + 2$

Step 5: Check with a known term

$n = 4$: $3 \times 4 + 2 = 12 + 2 = 14$ ✓

ANSWER nth term = $3n + 2$

Example 3.2 [2 marks]

Find the nth term of the sequence 7, 11, 15, 19, ...

Step 1: Find the common difference

$11 - 7 = 4$

$15 - 11 = 4$

$19 - 15 = 4$

Common difference = 4

Step 2: Start the rule with the difference times *n*

The nth term begins $4n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 7 - 4 = 3$

Step 4: Write the nth term

nth term = $4n + 3$

Step 5: Check with a known term

$n = 4$: $4 \times 4 + 3 = 16 + 3 = 19$ ✓

ANSWER nth term = $4n + 3$

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Example 3.3 [2 marks]

Find the nth term of the sequence 2, 9, 16, 23, ...

Step 1: Find the common difference

$9 - 2 = 7$

$16 - 9 = 7$

$23 - 16 = 7$

Common difference = 7

Step 2: Start the rule with the difference times *n*

The nth term begins $7n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 2 - 7 = -5$

Step 4: Write the nth term

nth term = $7n - 5$

Step 5: Check with a known term

$n = 4$: $7 \times 4 - 5 = 28 - 5 = 23$ ✓

ANSWER nth term = $7n - 5$

Example 3.4 [2 marks]

Find the nth term of the sequence 20, 17, 14, 11, ...

Step 1: Find the common difference

$17 - 20 = -3$

$14 - 17 = -3$

$11 - 14 = -3$

Common difference = −3

Step 2: Start the rule with the difference times *n*

The nth term begins $-3n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 20 - (-3) = 23$

Step 4: Write the nth term

nth term = $-3n + 23 = 23 - 3n$

Step 5: Check with a known term

$n = 4$: $23 - 3 \times 4 = 23 - 12 = 11$ ✓

ANSWER nth term = $23 - 3n$

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Example 3.5 [2 marks]

Find the nth term of the sequence −4, 2, 8, 14, …

Step 1: Find the common difference

$2 - (-4) = 6$

$8 - 2 = 6$

$14 - 8 = 6$

Common difference = 6

Step 2: Start the rule with the difference times *n*

The nth term begins $6n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= -4 - 6 = -10$

Step 4: Write the nth term

nth term = $6n - 10$

Step 5: Check with a known term

$n = 4$: $6 \times 4 - 10 = 24 - 10 = 14$ ✓

ANSWER nth term = $6n - 10$

Example 3.6 [4 marks]

Matchstick squares are arranged in rows. Pattern 1 uses 4 matchsticks, pattern 2 uses 7 and pattern 3 uses 10.

(a) Find the nth term. (b) How many matchsticks are in pattern 20?



Pattern 1
4 sticks

Pattern 2
7 sticks

Pattern 3
10 sticks

Step 1: Find the common difference

$7 - 4 = 3$

$10 - 7 = 3$

Common difference = 3

Step 2: Start the rule with the difference times *n*

The nth term begins $3n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 4 - 3 = 1$

Step 4: Write the nth term

nth term = $3n + 1$

Step 5: Check with a known term

$n = 3$: $3 \times 3 + 1 = 9 + 1 = 10$ ✓

Step 6: Substitute $n = 20$

Pattern 20 = $3 \times 20 + 1 = 60 + 1 = 61$

ANSWER (a) $3n + 1$ (b) 61 matchsticks

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Practice questions

Question 3.1 [2 marks]

Find the nth term of the sequence 3, 7, 11, 15, ...

Question 3.2 [2 marks]

Find the nth term of the sequence 10, 12, 14, 16, ...

Question 3.3 [2 marks]

Find the nth term of the sequence 1, 6, 11, 16, ...

Question 3.4 [2 marks]

Find the nth term of the sequence 40, 35, 30, 25, ...

Question 3.5 [2 marks]

Find the nth term of the sequence −7, −4, −1, 2, ...

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CONCEPT 4

Concept 4 || Is a number in the sequence?

04

Key idea

To test whether a number belongs to a sequence, set the nth term equal to that number and **solve the equation**.

If *n* comes out as a **whole number**, the number is in the sequence, and *n* tells you its position.

If *n* is a decimal or negative, the number is **not** in the sequence: there is no 24.25th term.

Method

1. Write nth term = the number.
2. Solve the equation for *n*.
3. Whole number: yes, it is the *n*th term.
4. Not a whole number: no, it is not a term.

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Worked examples

Example 4.1 [2 marks]

Is 47 a term in the sequence with nth term $3n + 2$?
If so, which term?

Step 1: Set the nth term equal to 47

$3n + 2 = 47$

Step 2: Subtract 2 from both sides

$3n = 47 - 2$

$3n = 45$

Step 3: Divide both sides by 3

$n = \frac{45}{3} = 15$

Step 4: Decide

$n = 15$ is a whole number, so 47 **is** a term. It is the 15th term.

ANSWER Yes, 47 is the 15th term

Example 4.2 [2 marks]

Is 100 a term in the sequence with nth term $4n + 3$?

Step 1: Set the nth term equal to 100

$4n + 3 = 100$

Step 2: Subtract 3 from both sides

$4n = 100 - 3$

$4n = 97$

Step 3: Divide both sides by 4

$n = \frac{97}{4} = 24.25$

Step 4: Decide

$n = 24.25$ is not a whole number, so 100 **is not** a term.

Check: 24th term $= 4 \times 24 + 3 = 96 + 3 = 99$

25th term $= 4 \times 25 + 3 = 100 + 3 = 103$

100 falls between these two terms ✓

ANSWER No, 100 is not a term

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Example 4.3 [3 marks]

Here is a sequence: 7, 13, 19, 25, ...
Is 121 a term in this sequence? Give a reason.

Step 1: Find the nth term of the sequence

Common difference = 13 − 7 = 6

Zeroth term = 7 − 6 = 1

nth term = $6n + 1$

Step 2: Set the nth term equal to 121

$6n + 1 = 121$

Step 3: Subtract 1 from both sides

$6n = 121 - 1$

$6n = 120$

Step 4: Divide both sides by 6

$$n = \frac{120}{6} = 20$$

Step 5: Decide

$n = 20$ is a whole number, so 121 **is** a term. It is the 20th term.

ANSWER Yes, 121 is the 20th term

Example 4.4 [3 marks]

Here is a sequence: 50, 46, 42, 38, ...
Will 0 ever appear in this sequence? Explain your answer.

Step 1: Find the nth term of the sequence

Common difference = 46 − 50 = −4

Zeroth term = 50 − (−4) = 54

nth term = $54 - 4n$

Step 2: Set the nth term equal to 0

$54 - 4n = 0$

Step 3: Add $4n$ to both sides, then subtract 0

$54 = 0 + 4n$

$4n = 54 - 0$

$4n = 54$

Step 4: Divide both sides by 4

$$n = \frac{54}{4} = 13.5$$

Step 5: Decide

$n = 13.5$ is not a whole number, so 0 **is not** a term.

Check: 13th term = $54 - 4 \times 13 = 54 - 52 = 2$

14th term = $54 - 4 \times 14 = 54 - 56 = -2$

0 falls between these two terms ✓

ANSWER No, 0 is never a term

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Example 4.5 [3 marks]

The nth term of a sequence is $7n - 3$. Find the first term that is greater than 100.

Step 1: Write an inequality: term greater than 100

$7n - 3 > 100$

Step 2: Add 3 on both sides

$7n > 100 + 3$

$7n > 103$

Step 3: Divide both sides by 7

$n > \frac{103}{7}$

$n > 14.7142857\ldots$

Step 4: Choose the smallest whole number

n must be a whole number, so the smallest value is $n = 15$

Step 5: Find that term

15th term $= 7 \times 15 - 3 = 105 - 3 = 102$

ANSWER The 15th term, 102

Example 4.6 [3 marks]

Here is a sequence: 5, 9, 13, 17, ...
Show that 85 is the 21st term of this sequence.

Step 1: Find the nth term of the sequence

Common difference $= 9 - 5 = 4$

Zeroth term $= 5 - 4 = 1$

nth term $= 4n + 1$

Step 2: Substitute $n = 21$ into the nth term

21st term $= 4 \times 21 + 1 = 84 + 1 = 85$

Step 3: Conclude

21st term = 85, as required ✓

ANSWER 85 is the 21st term ✓

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Practice questions

Question 4.1 [2 marks]

Is 62 a term in the sequence with nth term $5n - 3$?
If so, which term?

Question 4.2 [2 marks]

Is 90 a term in the sequence with nth term $4n + 3$?

Question 4.3 [3 marks]

Here is a sequence: 8, 15, 22, 29, ...
Is 150 a term in this sequence? Give a reason.

Question 4.4 [3 marks]

Here is a sequence: 60, 57, 54, 51, ...
Which term of the sequence is 3?

Question 4.5 [3 marks]

The nth term of a sequence is $9n + 4$. Find the first term that is greater than 200.

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CONCEPT 5

Concept 5 || Special sequences

05

Key idea

Some sequences are not linear but are so common that they have names.

Square numbers (n^2): 1, 4, 9, 16, 25, ... **Cube numbers** (n^3): 1, 8, 27, 64, 125, ...

Triangular numbers: 1, 3, 6, 10, 15, ... **Fibonacci sequence**: 1, 1, 2, 3, 5, 8, ... where each term is the sum of the two before it.

Method

1. Check the differences first. If they are not equal, the sequence is not linear.
2. Compare the terms with the square or cube numbers.
3. For Fibonacci type sequences, add the two previous terms.

Example table

Name	First five terms	Rule
Square	1, 4, 9, 16, 25	n^2
Cube	1, 8, 27, 64, 125	n^3
Triangular	1, 3, 6, 10, 15	add 1 more each time
Fibonacci	1, 1, 2, 3, 5	add the two previous

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Worked examples

Example 5.1 [2 marks]

Write down the first six square numbers and the nth term of the square numbers.

Step 1: Square each position number

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$
$6^2 = 6 \times 6 = 36$

Step 2: Write the rule

Each term is its position squared, so nth term = n^2

ANSWER 1, 4, 9, 16, 25, 36; nth term = n^2

Example 5.2 [2 marks]

Write down the first five cube numbers and the nth term of the cube numbers.

Step 1: Cube each position number

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$

Step 2: Write the rule

nth term = n^3

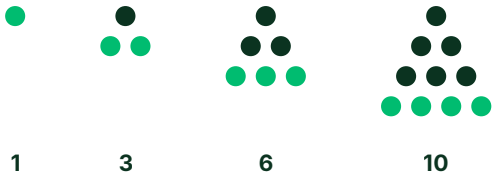
ANSWER 1, 8, 27, 64, 125; nth term = n^3

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Example 5.3

[4 marks]

The dot patterns show the triangular numbers. (a) Write down the first six triangular numbers. (b) Use the rule to find the 10th triangular number.



Step 1: Look at how each pattern grows

Each new pattern adds a row with one more dot than the last row.

Step 2: Add 2, then 3, then 4, then 5, then 6

1

1 + 2 = 3

3 + 3 = 6

6 + 4 = 10

10 + 5 = 15

15 + 6 = 21

Step 3: Use the rule for the nth triangular number

$$\text{nth triangular number} = \frac{n(n+1)}{2}$$

Step 4: Substitute $n = 10$

$$\frac{10 \times (10+1)}{2} = \frac{10 \times 11}{2} = \frac{110}{2} = 55$$

ANSWER (a) 1, 3, 6, 10, 15, 21 (b) 55

Example 5.4

[2 marks]

The Fibonacci sequence starts 1, 1, … Each term is the sum of the two terms before it. Write down the first eight terms.

Step 1: Start with the first two terms

1, 1

Step 2: Add the two previous terms each time

1 + 1 = 2

1 + 2 = 3

2 + 3 = 5

3 + 5 = 8

5 + 8 = 13

8 + 13 = 21

ANSWER 1, 1, 2, 3, 5, 8, 13, 21

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Example 5.5 [2 marks]

Here is a Fibonacci type sequence: 3, 4, 7, 11, ___, ___

Find the next two terms.

Step 1: Check the rule with the given terms

$3 + 4 = 7$ ✓

$4 + 7 = 11$ ✓

Step 2: Add the two previous terms

$7 + 11 = 18$

$11 + 18 = 29$

ANSWER 18, 29

Example 5.6 [3 marks]

Find the nth term of the sequence 2, 5, 10, 17, 26, ...

Step 1: Check the differences

$5 - 2 = 3$, $10 - 5 = 5$, $17 - 10 = 7$, $26 - 17 = 9$

The differences are not the same, so the sequence is not linear.

Step 2: Compare with the square numbers

Square numbers: 1, 4, 9, 16, 25

Sequence: 2, 5, 10, 17, 26

Each term is 1 more than the square number.

Step 3: Write the rule

nth term = $n^2 + 1$

Step 4: Check with a known term

$n = 4$: $4^2 + 1 = 16 + 1 = 17$ ✓

ANSWER nth term = $n^2 + 1$

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Practice questions

Question 5.1 [1 mark]

Write down the 7th, 8th and 9th square numbers.

Question 5.2 [2 marks]

Here are the first five triangular numbers: 1, 3, 6, 10, 15
Write down the next two triangular numbers.

Question 5.3 [2 marks]

Here is a Fibonacci type sequence: 2, 5, 7, 12, ___, ___
Find the next two terms.

Question 5.4 [2 marks]

Find the nth term of the sequence 0, 3, 8, 15, 24, ...

Question 5.5 [2 marks]

Show that 64 is both a square number and a cube number.

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Answer Key

Answer Summary for all 25 practice questions, followed by full worked solutions.

Q	Final answer
Concept 1 || Term to term rules	
1.1	9, 13, 17, 21, 25
1.2	100, 85, 70, 55, 40
1.3	Rule: add 8. Next two terms: 45, 53
1.4	Rule: subtract 4. Next two terms: −10, −14
1.5	11 and 23
Concept 2 || Position to term rules	
2.1	7, 9, 11, 13, 15
2.2	20th term = 119
2.3	26, 22, 18, 14
2.4	0, 3, 8, 15
2.5	First three terms: −3, 4, 11. 100th term: 690
Concept 3 || Finding the nth term	
3.1	nth term = $4n - 1$
3.2	nth term = $2n + 8$
3.3	nth term = $5n - 4$
3.4	nth term = $45 - 5n$
3.5	nth term = $3n - 10$
Concept 4 || Is a number in the sequence?	
4.1	Yes, 62 is the 13th term
4.2	No, 90 is not a term
4.3	No, 150 is not a term
4.4	3 is the 20th term
4.5	The 22nd term, 202
Concept 5 || Special sequences	
5.1	49, 64, 81
5.2	21, 28
5.3	19, 31
5.4	nth term = $n^2 - 1$
5.5	$64 = 8^2 = 4^3$ ✓

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Full worked solutions

Question 1.1 [2 marks]

A sequence starts at 9. The term to term rule is **add 4**. Write down the first five terms.

Step 1: Write down the first term

1st term = 9

Step 2: Apply the rule (add 4) to each term in turn

2nd term = 9 + 4 = 13
3rd term = 13 + 4 = 17
4th term = 17 + 4 = 21
5th term = 21 + 4 = 25

ANSWER 9, 13, 17, 21, 25

Question 1.2 [2 marks]

A sequence starts at 100. The term to term rule is **subtract 15**. Write down the first five terms.

Step 1: Write down the first term

1st term = 100

Step 2: Apply the rule (subtract 15) to each term in turn

2nd term = 100 − 15 = 85
3rd term = 85 − 15 = 70
4th term = 70 − 15 = 55
5th term = 55 − 15 = 40

ANSWER 100, 85, 70, 55, 40

Question 1.3 [3 marks]

Here is a sequence: 13, 21, 29, 37, ...
Find the term to term rule and the next two terms.

Step 1: Find the difference between consecutive terms

21 − 13 = 8
29 − 21 = 8
37 − 29 = 8

Step 2: State the term to term rule

The difference is always 8, so the rule is **add 8**.

Step 3: Apply the rule to find the next terms

37 + 8 = 45
45 + 8 = 53

ANSWER Rule: add 8. Next two terms: 45, 53

Question 1.4 [3 marks]

Here is a sequence: 6, 2, −2, −6, ...
Find the term to term rule and the next two terms.

Step 1: Find the difference between consecutive terms

2 − 6 = −4
(−2) − 2 = −4
(−6) − (−2) = −4

Step 2: State the term to term rule

The difference is always −4, so the rule is **subtract 4**.

Step 3: Apply the rule to find the next terms

−6 − 4 = −10
−10 − 4 = −14

ANSWER Rule: subtract 4. Next two terms: −10, −14

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Question 1.5 [3 marks]

The terms of this linear sequence go up by the same amount each time. Find the missing terms.

5, ___, 17, ___, 29

Step 1: Find the total change across two steps

17 − 5 = 12

Step 2: Divide by the number of steps

Difference = $\frac{12}{2}$ = 6

Step 3: Fill in the missing terms

2nd term = 5 + 6 = 11

4th term = 17 + 6 = 23

Step 4: Check with the last term

23 + 6 = 29 ✓

ANSWER 11 and 23

Question 2.1 [2 marks]

The nth term of a sequence is $2n + 5$. Write down the first five terms.

Step 1: Write down the nth term rule

nth term = $2n + 5$

Step 2: Substitute each position number for n

$n = 1$: $2 \times 1 + 5 = 2 + 5 = 7$

$n = 2$: $2 \times 2 + 5 = 4 + 5 = 9$

$n = 3$: $2 \times 3 + 5 = 6 + 5 = 11$

$n = 4$: $2 \times 4 + 5 = 8 + 5 = 13$

$n = 5$: $2 \times 5 + 5 = 10 + 5 = 15$

ANSWER 7, 9, 11, 13, 15

Question 2.2 [2 marks]

The nth term of a sequence is $6n - 1$. Find the 20th term.

Step 1: Write down the nth term rule

nth term = $6n - 1$

Step 2: Substitute each position number for n

$n = 20$: $6 \times 20 - 1 = 120 - 1 = 119$

ANSWER 20th term = 119

Question 2.3 [2 marks]

The nth term of a sequence is $30 - 4n$. Write down the first four terms.

Step 1: Write down the nth term rule

nth term = $30 - 4n$

Step 2: Substitute each position number for n

$n = 1$: $30 - 4 \times 1 = 30 - 4 = 26$

$n = 2$: $30 - 4 \times 2 = 30 - 8 = 22$

$n = 3$: $30 - 4 \times 3 = 30 - 12 = 18$

$n = 4$: $30 - 4 \times 4 = 30 - 16 = 14$

ANSWER 26, 22, 18, 14

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Question 2.4 [2 marks]

The nth term of a sequence is $n^2 - 1$. Write down the first four terms.

Step 1: Write down the nth term rule

nth term = $n^2 - 1$

Step 2: Substitute each position number for n

$n = 1$: $1^2 - 1 = 1 - 1 = 0$

$n = 2$: $2^2 - 1 = 4 - 1 = 3$

$n = 3$: $3^2 - 1 = 9 - 1 = 8$

$n = 4$: $4^2 - 1 = 16 - 1 = 15$

ANSWER 0, 3, 8, 15

Question 2.5 [3 marks]

The nth term of a sequence is $7n - 10$. Find the first three terms and the 100th term.

Step 1: Write down the nth term rule

nth term = $7n - 10$

Step 2: Substitute each position number for n

$n = 1$: $7 \times 1 - 10 = 7 - 10 = -3$

$n = 2$: $7 \times 2 - 10 = 14 - 10 = 4$

$n = 3$: $7 \times 3 - 10 = 21 - 10 = 11$

$n = 100$: $7 \times 100 - 10 = 700 - 10 = 690$

ANSWER First three terms: −3, 4, 11.
100th term: 690

Question 3.1 [2 marks]

Find the nth term of the sequence 3, 7, 11, 15, ...

Step 1: Find the common difference

$7 - 3 = 4$

$11 - 7 = 4$

$15 - 11 = 4$

Common difference = 4

Step 2: Start the rule with the difference times n

The nth term begins $4n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 3 - 4 = -1$

Step 4: Write the nth term

nth term = $4n - 1$

Step 5: Check with a known term

$n = 4$: $4 \times 4 - 1 = 16 - 1 = 15$ ✓

ANSWER nth term = $4n - 1$

Question 3.2 [2 marks]

Find the nth term of the sequence 10, 12, 14, 16, ...

Step 1: Find the common difference

$12 - 10 = 2$

$14 - 12 = 2$

$16 - 14 = 2$

Common difference = 2

Step 2: Start the rule with the difference times n

The nth term begins $2n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 10 - 2 = 8$

Step 4: Write the nth term

nth term = $2n + 8$

Step 5: Check with a known term

$n = 4$: $2 \times 4 + 8 = 8 + 8 = 16$ ✓

ANSWER nth term = $2n + 8$

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Question 3.3 [2 marks]

Find the nth term of the sequence 1, 6, 11, 16, ...

Step 1: Find the common difference

$6 - 1 = 5$

$11 - 6 = 5$

$16 - 11 = 5$

Common difference = 5

Step 2: Start the rule with the difference times *n*

The nth term begins $5n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 1 - 5 = -4$

Step 4: Write the nth term

nth term = $5n - 4$

Step 5: Check with a known term

$n = 4$: $5 \times 4 - 4 = 20 - 4 = 16$ ✓

ANSWER nth term = $5n - 4$

Question 3.4 [2 marks]

Find the nth term of the sequence 40, 35, 30, 25, ...

Step 1: Find the common difference

$35 - 40 = -5$

$30 - 35 = -5$

$25 - 30 = -5$

Common difference = −5

Step 2: Start the rule with the difference times *n*

The nth term begins $-5n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= 40 - (-5) = 45$

Step 4: Write the nth term

nth term = $-5n + 45 = 45 - 5n$

Step 5: Check with a known term

$n = 4$: $45 - 5 \times 4 = 45 - 20 = 25$ ✓

ANSWER nth term = $45 - 5n$

Question 3.5 [2 marks]

Find the nth term of the sequence −7, −4, −1, 2, ...

Step 1: Find the common difference

$(-4) - (-7) = 3$

$(-1) - (-4) = 3$

$2 - (-1) = 3$

Common difference = 3

Step 2: Start the rule with the difference times *n*

The nth term begins $3n$

Step 3: Find the zeroth term (the term before the first)

Zeroth term = 1st term − difference

$= -7 - 3 = -10$

Step 4: Write the nth term

nth term = $3n - 10$

Step 5: Check with a known term

$n = 4$: $3 \times 4 - 10 = 12 - 10 = 2$ ✓

ANSWER nth term = $3n - 10$

Question 4.1 [2 marks]

Is 62 a term in the sequence with nth term $5n - 3$? If so, which term?

Step 1: Set the nth term equal to 62

$5n - 3 = 62$

Step 2: Add 3 to both sides

$5n = 62 + 3$

$5n = 65$

Step 3: Divide both sides by 5

$$n = \frac{65}{5} = 13$$

Step 4: Decide

$n = 13$ is a whole number, so 62 **is** a term. It is the 13th term.

ANSWER Yes, 62 is the 13th term

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Question 4.2 [2 marks]

Is 90 a term in the sequence with nth term $4n + 3$?

Step 1: Set the nth term equal to 90

$4n + 3 = 90$

Step 2: Subtract 3 from both sides

$4n = 90 - 3$

$4n = 87$

Step 3: Divide both sides by 4

$n = \frac{87}{4} = 21.75$

Step 4: Decide

$n = 21.75$ is not a whole number, so 90 **is not** a term.

Check: 21st term $= 4 \times 21 + 3 = 84 + 3 = 87$

22nd term $= 4 \times 22 + 3 = 88 + 3 = 91$

90 falls between these two terms ✓

ANSWER No, 90 is not a term

Question 4.3 [3 marks]

Here is a sequence: 8, 15, 22, 29, ...

Is 150 a term in this sequence? Give a reason.

Step 1: Find the nth term of the sequence

Common difference $= 15 - 8 = 7$

Zeroth term $= 8 - 7 = 1$

nth term $= 7n + 1$

Step 2: Set the nth term equal to 150

$7n + 1 = 150$

Step 3: Subtract 1 from both sides

$7n = 150 - 1$

$7n = 149$

Step 4: Divide both sides by 7

$n = \frac{149}{7} = 21.2857142...$

Step 5: Decide

$n = 21.2857142...$ is not a whole number, so 150 **is not** a term.

Check: 21st term $= 7 \times 21 + 1 = 147 + 1 = 148$

22nd term $= 7 \times 22 + 1 = 154 + 1 = 155$

150 falls between these two terms ✓

ANSWER No, 150 is not a term

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Question 4.4 [3 marks]

Here is a sequence: 60, 57, 54, 51, ...
Which term of the sequence is 3?

Step 1: Find the nth term of the sequence

Common difference = 57 − 60 = −3

Zeroth term = 60 − (−3) = 63

nth term = $63 - 3n$

Step 2: Set the nth term equal to 3

$63 - 3n = 3$

Step 3: Add $3n$ to both sides, then subtract 3

$63 = 3 + 3n$

$3n = 63 - 3$

$3n = 60$

Step 4: Divide both sides by 3

$n = \frac{60}{3} = 20$

Step 5: Decide

$n = 20$ is a whole number, so 3 **is** a term. It is the 20th term.

ANSWER 3 is the 20th term

Question 4.5 [3 marks]

The nth term of a sequence is $9n + 4$. Find the first term that is greater than 200.

Step 1: Write an inequality: term greater than 200

$9n + 4 > 200$

Step 2: Subtract 4 on both sides

$9n > 200 - 4$

$9n > 196$

Step 3: Divide both sides by 9

$n > \frac{196}{9}$

$n > 21.7777777...$

Step 4: Choose the smallest whole number

n must be a whole number, so the smallest value is $n = 22$

Step 5: Find that term

22nd term = 9 × 22 + 4 = 198 + 4 = 202

ANSWER The 22nd term, 202

Question 5.1 [1 mark]

Write down the 7th, 8th and 9th square numbers.

Step 1: Square each position number

$7^2 = 7 \times 7 = 49$

$8^2 = 8 \times 8 = 64$

$9^2 = 9 \times 9 = 81$

ANSWER 49, 64, 81

Question 5.2 [2 marks]

Here are the first five triangular numbers: 1, 3, 6, 10, 15
Write down the next two triangular numbers.

Step 1: Find the pattern in the differences

$3 - 1 = 2, 6 - 3 = 3, 10 - 6 = 4, 15 - 10 = 5$

The difference goes up by 1 each time.

Step 2: Continue the pattern

$15 + 6 = 21$

$21 + 7 = 28$

ANSWER 21, 28

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Question 5.3 [2 marks]

Here is a Fibonacci type sequence: 2, 5, 7, 12, ___, ___

Find the next two terms.

Step 1: Check the rule with the given terms

$2 + 5 = 7$ ✓

$5 + 7 = 12$ ✓

Step 2: Add the two previous terms

$7 + 12 = 19$

$12 + 19 = 31$

ANSWER 19, 31

Question 5.4 [2 marks]

Find the nth term of the sequence 0, 3, 8, 15, 24, ...

Step 1: Check the differences

$3 - 0 = 3,\ 8 - 3 = 5,\ 15 - 8 = 7,\ 24 - 15 = 9$

Not linear: the differences change.

Step 2: Compare with the square numbers

Square numbers: 1, 4, 9, 16, 25

Each term is 1 less than the square number.

Step 3: Write the rule

nth term = $n^2 - 1$

Step 4: Check with a known term

$n = 4$: $4^2 - 1 = 16 - 1 = 15$ ✓

ANSWER nth term = $n^2 - 1$

Question 5.5 [2 marks]

Show that 64 is both a square number and a cube number.

Step 1: Write 64 as a square

$8^2 = 8 \times 8 = 64$

Step 2: Write 64 as a cube

$4^3 = 4 \times 4 \times 4 = 16 \times 4 = 64$

Step 3: Conclude

$64 = 8^2$ and $64 = 4^3$, so 64 is both a square number and a cube number, as required ✓

ANSWER $64 = 8^2 = 4^3$ ✓

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