Sismo Learning

YEAR 9 || NUMBER AND FINANCIAL MATHS

Compound Interest

Lesson Note || Growth, depreciation and working backwards

Inside this note

1. Percentage multipliers
2. Simple interest (for comparison)
3. Compound interest year by year
4. The compound interest formula
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7. Working backwards: finding the original amount
8. Finding the number of years
9. Finding the interest rate
10. Mixed exam practice
11. Formula summary
12. Answer key

By the end of this lesson you will be able to

1. Write a multiplier for any percentage increase or decrease.
2. Tell the difference between simple and compound interest, and compare them.
3. Calculate compound interest year by year and with the formula $A = P \times (1 + \frac{r}{100})^n$.
4. Solve depreciation, population growth and decay problems.
5. Handle interest added half yearly, quarterly or monthly.
6. Work backwards to find the original amount, the number of years or the rate.

Every concept contains a full explanation, six worked examples and six practice questions. Full worked solutions are in the answer key at the back.

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BEFORE YOU START

Key words and prior knowledge

Principal (P): the amount of money you start with.

Interest: extra money paid by a bank for keeping your money, or charged by a lender when you borrow.

Rate (r): the percentage of interest paid each year. **Per annum (p.a.)** means per year.

Simple interest: interest worked out on the original amount only, so it is the same every year.

Compound interest: interest worked out on the current balance, so you earn interest on your interest.

Balance: the amount in the account at a given time.

Multiplier: a single number that does a percentage change in one multiplication, for example 1.05 for a 5% increase.

Depreciation: a fall in value over time, for example a car losing value each year.

Appreciation: a rise in value over time, for example a house becoming worth more.

Compounded half yearly / quarterly / monthly: interest is added 2, 4 or 12 times every year.

PRIOR KNOWLEDGE CHECKLIST

- Converting percentages to decimals: $35\% = \frac{35}{100} = 0.35$ and $4.5\% = \frac{4.5}{100} = 0.045$
- Finding a percentage of an amount: 15% of £40 = 0.15×40 = £6
- Using the power key on a calculator: $1.05^3 = 1.157625$
- Rounding money to the nearest penny (2 decimal places)

How to read the worked examples

Every worked example is laid out exactly as you should write it in your book: the question first, then numbered steps, each with a short title saying what you are doing, then every line of working, and finally the answer in a green ANSWER box. Full calculator values are kept (shown with ...) and only rounded in the final step.

When we rearrange an equation we use the **carry over** method: when a term crosses the equals sign, its operation changes. Multiplying becomes dividing, adding becomes subtracting, and a power becomes a root.

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CONCEPT 1

Percentage multipliers

Every compound interest question is built on one idea: **a percentage change can be done in a single multiplication**. The number you multiply by is called the **multiplier**.

The original amount is always 100%. If something increases by 6%, it becomes 100% + 6% = 106% of what it was. As a decimal, $106\% = \frac{106}{100} = 1.06$. So multiplying by 1.06 increases any amount by 6% in one step.

If something decreases by 15%, it becomes 100% − 15% = 85% of what it was, and $85\% = \frac{85}{100} = 0.85$.

KEY FACT

Increase by r%	multiplier = $1 + \frac{r}{100}$	always **bigger** than 1
Decrease by r%	multiplier = $1 - \frac{r}{100}$	always **smaller** than 1

Change	Percentage left	Multiplier
Increase 5%	105%	1.05
Increase 12%	112%	1.12
Increase 3.5%	103.5%	1.035
Decrease 20%	80%	0.8
Decrease 7%	93%	0.93
Decrease 0.5%	99.5%	0.995

WATCH OUT

- A 5% increase is 1.05, **not** 1.5. 1.5 would be a 50% increase.
- A 7% decrease is 0.93, **not** 0.07. 0.07 only gives you the amount taken off.
- Check your multiplier: an increase must give a number above 1, a decrease a number below 1.

Worked examples || Percentage multipliers

Example 1.1 Write down the multiplier for a increase of 6%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 6% gives 100% + 6% = 106%

Step 2: Change the percentage into a decimal

$106\% = \frac{106}{100} = 1.06$

ANSWER **Multiplier = 1.06**

Example 1.2 Write down the multiplier for a decrease of 15%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A decrease of 15% gives 100% − 15% = 85%

Step 2: Change the percentage into a decimal

$85\% = \frac{85}{100} = 0.85$

ANSWER **Multiplier = 0.85**

Example 1.3 Write down the multiplier for a increase of 2.5%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 2.5% gives 100% + 2.5% = 102.5%

Step 2: Change the percentage into a decimal

$102.5\% = \frac{102.5}{100} = 1.025$

ANSWER **Multiplier = 1.025**

Example 1.4 Increase £240 by 15% using a multiplier. **[2 marks]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 15% gives 100% + 15% = 115%

Step 2: Change the percentage into a decimal

$115\% = \frac{115}{100} = 1.15$

Step 3: Multiply the original amount by the multiplier

£240 × 1.15 = £276.00

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ANSWER £276.00

Example 1.5 Decrease £86 by 30% using a multiplier. [2 marks]

Step 1: Start from 100%

The original amount is always 100%.

A decrease of 30% gives 100% − 30% = 70%

Step 2: Change the percentage into a decimal

$70\% = \frac{70}{100} = 0.7$

Step 3: Multiply the original amount by the multiplier

£86 × 0.7 = £60.20

ANSWER £60.20

Example 1.6 Increase £1,250 by 3.5% using a multiplier. [2 marks]

Step 1: Start from 100%

The original amount is always 100%.

A increase of 3.5% gives 100% + 3.5% = 103.5%

Step 2: Change the percentage into a decimal

$103.5\% = \frac{103.5}{100} = 1.035$

Step 3: Multiply the original amount by the multiplier

£1,250 × 1.035 = £1,293.75

ANSWER £1,293.75

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Practice questions || Percentage multipliers

Show all your working. Answers are in the answer key at the back.

Q1.1 Write down the multiplier for a increase of 9%. **[1 mark]**

Q1.2 Write down the multiplier for a decrease of 22%. **[1 mark]**

Q1.3 Write down the multiplier for a increase of 0.5%. **[1 mark]**

Q1.4 Increase £560 by 12% using a multiplier. **[2 marks]**

Q1.5 Decrease £45 by 8% using a multiplier. **[2 marks]**

Q1.6 Decrease £3,200 by 4.5% using a multiplier. **[2 marks]**

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CONCEPT 2

Simple interest (for comparison)

When you put money in a bank, the bank pays you extra money called **interest**. The money you start with is called the **principal** (P). The interest rate is usually given **per annum**, which means per year.

With **simple interest**, the interest is always worked out on the **original amount only**. So you earn exactly the same interest every year.

KEY FORMULA

Interest for one year = $\frac{r}{100} \times P$

Simple interest $I = \frac{P \times r \times n}{100}$

Total amount = $P + I$

Simple interest is not the main topic of this lesson, but many exam questions ask you to **compare** it with compound interest, so you must be able to do both.

WATCH OUT

- Read the question: does it want the **interest** only, or the **total amount** (principal + interest)?
- Simple interest never changes from year to year. If your yearly interest is growing, you are doing compound interest.

Worked examples || Simple interest (for comparison)

Example 2.1 £800 is invested at 5% per year **simple** interest for 3 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

5% of £800 = $\frac{5}{100} \times 800$

= £40.00

Step 2: Multiply by the number of years

£40.00 × 3 = £120.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£800.00 + £120.00 = £920.00

ANSWER **Total = £920.00**

Example 2.2 £2,500 is invested at 4% per year **simple** interest for 6 years. Work out the interest earned. **[2 marks]**

Step 1: Find the interest for one year

4% of £2,500 = $\frac{4}{100} \times 2{,}500$

= £100.00

Step 2: Multiply by the number of years

£100.00 × 6 = £600.00

Simple interest is the same every year because it is always worked out on the original amount.

ANSWER **Interest = £600.00**

Example 2.3 £1,200 is invested at 3.5% per year **simple** interest for 4 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

3.5% of £1,200 = $\frac{3.5}{100} \times 1{,}200$

= £42.00

Step 2: Multiply by the number of years

£42.00 × 4 = £168.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£1,200.00 + £168.00 = £1,368.00

ANSWER **Total = £1,368.00**

Example 2.4 £450 is invested at 2% per year **simple** interest for 7 years. Work out the interest earned. **[2 marks]**

Step 1: Find the interest for one year

2% of £450 = $\frac{2}{100} \times 450$

= £9.00

Step 2: Multiply by the number of years

£9.00 × 7 = £63.00

Simple interest is the same every year because it is always worked out on the original amount.

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ANSWER **Interest = £63.00**

Example 2.5 £15,000 is invested at 6% per year **simple** interest for 2 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

6% of £15,000 = $\frac{6}{100} \times 15{,}000$

= £900.00

Step 2: Multiply by the number of years

£900.00 × 2 = £1,800.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£15,000.00 + £1,800.00 = £16,800.00

ANSWER **Total = £16,800.00**

Example 2.6 £640 is invested at 7.5% per year **simple** interest for 5 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

7.5% of £640 = $\frac{7.5}{100} \times 640$

= £48.00

Step 2: Multiply by the number of years

£48.00 × 5 = £240.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£640.00 + £240.00 = £880.00

ANSWER **Total = £880.00**

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Practice questions || Simple interest (for comparison)

Show all your working. Answers are in the answer key at the back.

Q2.1 £300 is invested at 4% per year **simple** interest for 5 years. Work out the interest earned. **[2 marks]**

Q2.2 £1,800 is invested at 6% per year **simple** interest for 3 years. Work out the total amount in the account at the end. **[2 marks]**

Q2.3 £950 is invested at 2.5% per year **simple** interest for 8 years. Work out the interest earned. **[2 marks]**

Q2.4 £5,200 is invested at 3% per year **simple** interest for 4 years. Work out the total amount in the account at the end. **[2 marks]**

Q2.5 £720 is invested at 8.5% per year **simple** interest for 2 years. Work out the interest earned. **[2 marks]**

Q2.6 £10,000 is invested at 1.5% per year **simple** interest for 10 years. Work out the total amount in the account at the end. **[2 marks]**

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CONCEPT 3

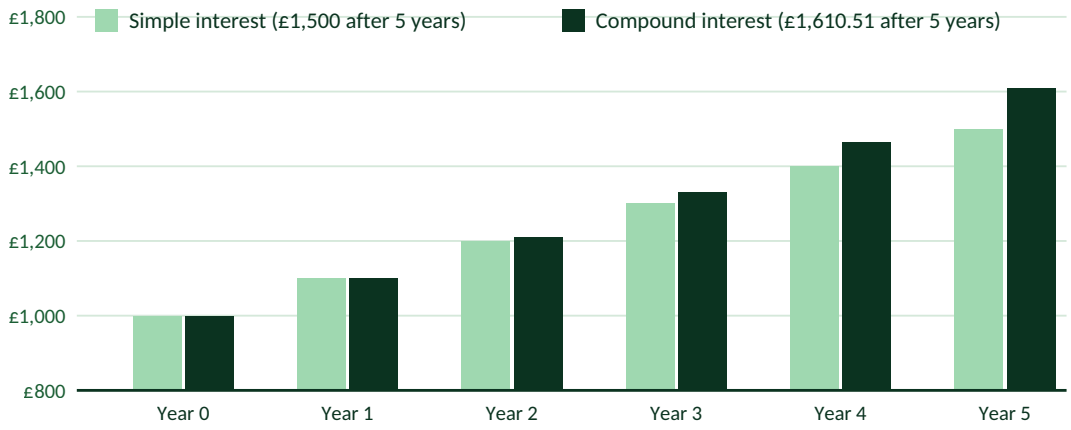
Compound interest year by year

With **compound interest**, the interest earned each year is **added to the balance**, and next year's interest is worked out on this new, bigger balance. You earn **interest on your interest**, so the amount grows faster every year.

The year by year method shows exactly what the bank does. For each year: work out the interest on the current balance, then add it on.

Year	Balance at start	Interest (10%)	Balance at end
1	£1,000.00	£100.00	£1,100.00
2	£1,100.00	£110.00	£1,210.00
3	£1,210.00	£121.00	£1,331.00

Notice the interest grows: £100, then £110, then £121. With simple interest it would stay at £100 every year. The chart shows £1,000 at 10% over 5 years.



Adding 10% is the same as multiplying by the multiplier 1.10, so each year you can simply multiply the balance by the multiplier.

> WATCH OUT
>
> - Do **not** work out the interest on the original amount every year. That is simple interest.
> - Keep full values (do not round) until the very last step, otherwise your answer can be a penny out.
> - Read whether the question wants the final **amount** or just the **interest**.

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Worked examples || Compound interest year by year

Example 3.1 £1,000 is invested at 10% per year **compound** interest. Working year by year, find the total amount after 2 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{10}{100} \times 1{,}000 = 100$

Balance = 1,000 + 100 = **1,100**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{10}{100} \times 1{,}100 = 110$

Balance = 1,100 + 110 = **1,210**

Step 3: Round to the nearest penny

Total after 2 years = £1,210



ANSWER **Total = £1,210.00**

Example 3.2 £500 is invested at 4% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{4}{100} \times 500 = 20$

Balance = 500 + 20 = **520**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{4}{100} \times 520 = 20.80$

Balance = 520 + 20.80 = **540.80**

Step 3: Year 3: find the interest and add it on

Interest = $\frac{4}{100} \times 540.80 = 21.632$

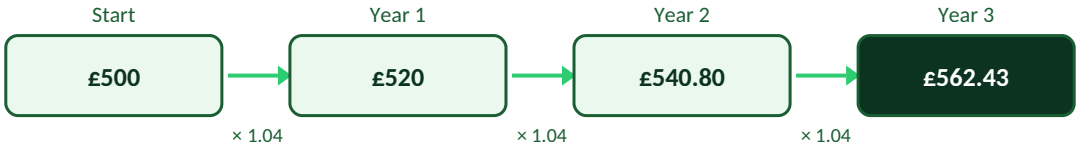
Balance = 540.80 + 21.632 = **562.432**

Step 4: Round to the nearest penny

Total after 3 years = £562.432

= £562.43 (2 d.p., to the nearest penny)

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ANSWER **Total = £562.43**

Example 3.3 £2,000 is invested at 5% per year **compound** interest. Working year by year, find the total interest earned after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{5}{100} \times 2{,}000 = 100$

Balance = 2,000 + 100 = **2,100**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{5}{100} \times 2{,}100 = 105$

Balance = 2,100 + 105 = **2,205**

Step 3: Year 3: find the interest and add it on

Interest = $\frac{5}{100} \times 2{,}205 = 110.25$

Balance = 2,205 + 110.25 = **2,315.25**

Step 4: Round to the nearest penny

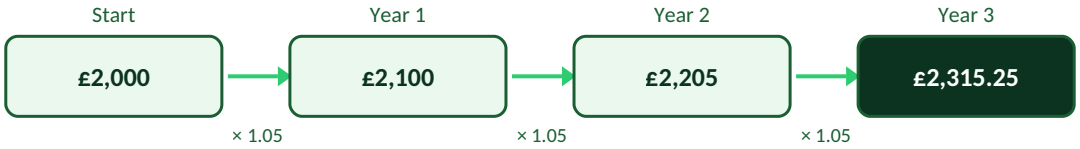
Total after 3 years = £2,315.25

Step 5: Find the total interest

Interest = final amount − original amount

= 2,315.25 − 2,000 = 315.25

= £315.25 (2 d.p.)



ANSWER **Interest = £315.25**

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Example 3.4 £800 is invested at 3% per year **compound** interest. Working year by year, find the total amount after 2 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{3}{100} \times 800 = 24$

Balance = 800 + 24 = **824**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{3}{100} \times 824 = 24.72$

Balance = 824 + 24.72 = **848.72**

Step 3: Round to the nearest penny

Total after 2 years = £848.72



ANSWER **Total = £848.72**

Example 3.5 £1,500 is invested at 6% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{6}{100} \times 1{,}500 = 90$

Balance = 1,500 + 90 = **1,590**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{6}{100} \times 1{,}590 = 95.40$

Balance = 1,590 + 95.40 = **1,685.40**

Step 3: Year 3: find the interest and add it on

Interest = $\frac{6}{100} \times 1{,}685.40 = 101.124$

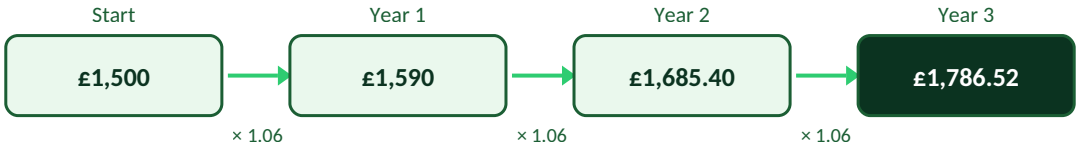
Balance = 1,685.40 + 101.124 = **1,786.524**

Step 4: Round to the nearest penny

Total after 3 years = £1,786.524

= £1,786.52 (2 d.p., to the nearest penny)

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ANSWER **Total = £1,786.52**

Example 3.6 £1,000 is invested for 3 years at 5% per year. How much **more** interest is earned with compound interest than with simple interest? **[4 marks]**

Step 1: Work out the simple interest

One year: $\frac{5}{100} \times 1{,}000 = 50$

3 years: $50 \times 3 = £150.00$

Step 2: Work out the compound amount

Multiplier = $1 + \frac{5}{100} = 1.05$

$1{,}000 \times (1.05)^3 = 1{,}000 \times 1.157625 = 1{,}157.625$

Step 3: Work out the compound interest

$1{,}157.625 - 1{,}000 = 157.625$

Step 4: Find the difference

$157.625 - 150 = 7.625$

$= £7.63$ (2 d.p.)

ANSWER **Compound earns £7.63 more**

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Practice questions || Compound interest year by year

Show all your working. Answers are in the answer key at the back.

Q3.1 £600 is invested at 10% per year **compound** interest. Working year by year, find the total amount after 2 years. **[3 marks]**

Q3.2 £400 is invested at 5% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Q3.3 £2,500 is invested at 2% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Q3.4 £1,200 is invested at 8% per year **compound** interest. Working year by year, find the total interest earned after 2 years. **[3 marks]**

Q3.5 £3,000 is invested at 4% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

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Q3.6 £700 is invested for 3 years at 6% per year. How much **more** interest is earned with compound interest than with simple interest? **[4 marks]**

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CONCEPT 4

The compound interest formula

Working year by year is fine for 2 or 3 years, but for 10 or 20 years it takes far too long. Because each year multiplies the balance by the same multiplier, we can do all the years at once using a **power**.

For example, £1,000 at 10% for 3 years: $1000 \times 1.1 \times 1.1 \times 1.1 = 1000 \times 1.1^3 =$ £1,331. This matches the table above.

KEY FORMULA

$$A = P \times (1 + \frac{r}{100})^n$$

- A final amount after n years
- P principal (the starting amount)
- r interest rate per year (as a percentage)
- n number of years

Compound interest earned = $A - P$

Using your calculator: type the multiplier, press the power key (x□ or ^), type the number of years, then multiply by the principal. Write down the full calculator display before rounding.

WATCH OUT

- The formula gives the **total amount**. If the question asks for the interest, you must take away the principal at the end.
- Work out the power **before** multiplying by {V("P")}. 1000 × {pw("1.1", 3)} is not the same as {pw("(1000 × 1.1)", 3)}.
- Round money to the nearest penny (2 d.p.) only at the very end.

Worked examples || The compound interest formula

Example 4.1 £3,000 is invested at 4% per year compound interest for 5 years. Work out the total amount. **[3 marks]**

Step 1: Write down the formula

$$A = P \times (1 + \frac{r}{100})^n$$

Step 2: Identify the values

P = 3,000 (starting amount), r = 4, n = 5

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Step 3: Work out the multiplier

$1 + \frac{4}{100} = 1 + 0.04 = 1.04$

Step 4: Substitute into the formula

$A = 3{,}000 \times (1.04)^5$

Step 5: Work out the power first

$(1.04)^5 = 1.2166\ldots$

Step 6: Multiply by the starting amount

$A = 3{,}000 \times 1.2166\ldots = 3{,}649.9587\ldots$

Step 7: Round to the nearest penny

$A = £3{,}649.96$ (2 d.p.)

ANSWER **Total = £3,649.96**

Example 4.2 £850 is invested at 2.5% compound interest per annum for 8 years. Find the value of the investment. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 850$ (starting amount), $r = 2.5$, $n = 8$

Step 3: Work out the multiplier

$1 + \frac{2.5}{100} = 1 + 0.025 = 1.025$

Step 4: Substitute into the formula

$A = 850 \times (1.025)^8$

Step 5: Work out the power first

$(1.025)^8 = 1.2184\ldots$

Step 6: Multiply by the starting amount

$A = 850 \times 1.2184\ldots = 1{,}035.6424\ldots$

Step 7: Round to the nearest penny

$A = £1{,}035.64$ (2 d.p.)

ANSWER **Total = £1,035.64**

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Example 4.3 £12,000 is saved at 3.2% per year compound interest for 6 years. How much interest is earned? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 12{,}000$ (starting amount), $r = 3.2$, $n = 6$

Step 3: Work out the multiplier

$1 + \frac{3.2}{100} = 1 + 0.032 = 1.032$

Step 4: Substitute into the formula

$A = 12{,}000 \times (1.032)^6$

Step 5: Work out the power first

$(1.032)^6 = 1.2080\ldots$

Step 6: Multiply by the starting amount

$A = 12{,}000 \times 1.2080\ldots = 14{,}496.3754\ldots$

Step 7: Round to the nearest penny

$A =$ £14,496.38 (2 d.p.)

Step 8: Find the interest

Interest = final amount − original amount

$= 14{,}496.3754\ldots - 12{,}000 = 2{,}496.3754\ldots$

= £2,496.38 (2 d.p.)

ANSWER **Interest = £2,496.38**

Example 4.4 £450 is invested for 10 years at 6% per year compound interest. Find the final amount and the interest earned. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 450$ (starting amount), $r = 6$, $n = 10$

Step 3: Work out the multiplier

$1 + \frac{6}{100} = 1 + 0.06 = 1.06$

Step 4: Substitute into the formula

$A = 450 \times (1.06)^{10}$

Step 5: Work out the power first

$(1.06)^{10} = 1.7908\ldots$

Step 6: Multiply by the starting amount

$A = 450 \times 1.7908\ldots = 805.8814\ldots$

Step 7: Round to the nearest penny

$A = £805.88$ (2 d.p.)

Step 8: Find the interest

Interest = final amount − original amount

$= 805.8814\ldots - 450 = 355.8814\ldots$

$= £355.88$ (2 d.p.)

ANSWER **Total = £805.88; Interest = £355.88**

Example 4.5 Show that £5,000 invested at 4% per year compound interest for 3 years grows to £5,624.32 **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 5{,}000$ (starting amount), $r = 4$, $n = 3$

Step 3: Work out the multiplier

$1 + \frac{4}{100} = 1 + 0.04 = 1.04$

Step 4: Substitute into the formula

$A = 5{,}000 \times (1.04)^3$

Step 5: Work out the power first

$(1.04)^3 = 1.124864$

Step 6: Multiply by the starting amount

$A = 5{,}000 \times 1.124864 = 5{,}624.32$

Step 7: Round to the nearest penny

$A = £5{,}624.32$ (2 d.p.)

$A =$ **£5,624.32** as required ✓

ANSWER **Shown: £5,624.32 as required ✓**

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Example 4.6 £20,000 is put in a savings bond paying 1.8% per year compound interest. Work out its value after 15 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

P = 20,000 (starting amount), r = 1.8, n = 15

Step 3: Work out the multiplier

$1 + \frac{1.8}{100} = 1 + 0.018 = 1.018$

Step 4: Substitute into the formula

$A = 20{,}000 \times (1.018)^{15}$

Step 5: Work out the power first

$(1.018)^{15} = 1.3068\ldots$

Step 6: Multiply by the starting amount

$A = 20{,}000 \times 1.3068\ldots = 26{,}136.4540\ldots$

Step 7: Round to the nearest penny

A = £26,136.45 (2 d.p.)

ANSWER **Total = £26,136.45**

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Practice questions || The compound interest formula

Show all your working. Answers are in the answer key at the back.

Q4.1 £2,400 is invested at 3% per year compound interest for 7 years. Work out the total amount. **[3 marks]**

Q4.2 £600 is invested at 5.5% per year compound interest for 4 years. How much interest is earned? **[3 marks]**

Q4.3 £15,000 is invested at 2.75% per annum compound interest. Find its value after 12 years. **[3 marks]**

Q4.4 £980 is invested at 7% per year compound interest for 6 years. Find the final amount and the interest earned. **[3 marks]**

Q4.5 Show that £8,000 invested at 2% per year compound interest for 4 years grows to £8,659.46 **[3 marks]**

Q4.6 A grandparent invests £350 for a newborn at 4.5% per year compound interest. How much is it worth after 20 years? **[3 marks]**

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CONCEPT 5

Depreciation and other growth and decay

Not everything grows. Cars, phones, laptops and machines lose value every year. This is called **depreciation**. When the value drops by the same percentage each year, it is **compound decay**, and we use a multiplier **less than 1**.

KEY FORMULA: DEPRECIATION

$$A = P \times (1 - \frac{r}{100})^n$$

Loss in value = $P - A$

The same formulas also work outside money. Any quantity that changes by the same percentage each period follows this pattern: populations of people or animals, bacteria, the number of visitors to a website, or the value of a house. When the answer is a count of people or animals, round to a **whole number**.

Growth (getting bigger)	Decay (getting smaller)
Multiplier above 1, for example 1.04	Multiplier below 1, for example 0.85
Savings, house prices, populations rising	Car values, phone values, populations falling

WATCH OUT

- A car losing 15% each year has multiplier 0.85, **not** 0.15.
- Depreciation never makes the value reach zero; it keeps getting smaller by the same fraction.
- A question that asks how much value is **lost** needs the subtraction at the end.

Worked examples || Depreciation and other growth and decay

Example 5.1 A car is bought for £18,000. It depreciates by 15% per year. Find its value after 3 years. **[3 marks]**

Step 1: Write down the formula

$$A = P \times (1 - \frac{r}{100})^n$$

Step 2: Identify the values

$P = 18{,}000$ (starting value), $r = 15$, $n = 3$

Step 3: Work out the multiplier

$$1 - \frac{15}{100} = 1 - 0.15 = 0.85$$

Step 4: Substitute into the formula

$$A = 18{,}000 \times (0.85)^3$$

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Step 5: Work out the power first

$(0.85)^3 = 0.614125$

Step 6: Multiply by the starting value

$A = 18{,}000 \times 0.614125 = 11{,}054.25$

Step 7: Round to the nearest penny

$A = £11{,}054.25$

ANSWER **Value = £11,054.25**

Example 5.2 A laptop costs £1,200 and loses 20% of its value each year. Work out its value after 4 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 1{,}200$ (starting value), $r = 20$, $n = 4$

Step 3: Work out the multiplier

$1 - \frac{20}{100} = 1 - 0.2 = 0.8$

Step 4: Substitute into the formula

$A = 1{,}200 \times (0.8)^4$

Step 5: Work out the power first

$(0.8)^4 = 0.4096$

Step 6: Multiply by the starting value

$A = 1{,}200 \times 0.4096 = 491.52$

Step 7: Round to the nearest penny

$A = £491.52$

ANSWER **Value = £491.52**

Example 5.3 A lake has 5,000 fish. The number of fish falls by 8% each year. Estimate the number of fish after 5 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

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Step 2: Identify the values

$P = 5{,}000$ (starting population), $r = 8$, $n = 5$

Step 3: Work out the multiplier

$1 - \frac{8}{100} = 1 - 0.08 = 0.92$

Step 4: Substitute into the formula

$A = 5{,}000 \times (0.92)^5$

Step 5: Work out the power first

$(0.92)^5 = 0.6590...$

Step 6: Multiply by the starting population

$A = 5{,}000 \times 0.6590... = 3{,}295.407616$

Step 7: Round to a whole number

$3{,}295.407616 \approx 3{,}295$

We cannot have part of a fish, so round to the nearest whole number.

ANSWER **3,295 fish**

Example 5.4 A phone costs £950 and depreciates by 30% per year. How much value has it lost after 2 years? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 950$ (starting value), $r = 30$, $n = 2$

Step 3: Work out the multiplier

$1 - \frac{30}{100} = 1 - 0.3 = 0.7$

Step 4: Substitute into the formula

$A = 950 \times (0.7)^2$

Step 5: Work out the power first

$(0.7)^2 = 0.49$

Step 6: Multiply by the starting value

$A = 950 \times 0.49 = 465.5$

Step 7: Round to the nearest penny

$A =$ £465.50

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Step 8: Find the loss in value

Loss = starting value − final value

= 950 − 465.5 = 484.5

= £484.50 (2 d.p.)

ANSWER **Loss in value = £484.50**

Example 5.5 A factory machine is worth £45,000. It depreciates by 12% per year. Find its value after 6 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 45{,}000$ (starting value), $r = 12$, $n = 6$

Step 3: Work out the multiplier

$1 - \frac{12}{100} = 1 - 0.12 = 0.88$

Step 4: Substitute into the formula

$A = 45{,}000 \times (0.88)^6$

Step 5: Work out the power first

$(0.88)^6 = 0.4644...$

Step 6: Multiply by the starting value

$A = 45{,}000 \times 0.4644... = 20{,}898.1839...$

Step 7: Round to the nearest penny

A = £20,898.18 (2 d.p.)

ANSWER **Value = £20,898.18**

Example 5.6 A town has 42,000 people. The population grows by 2% each year. Estimate the population after 5 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 42{,}000$ (starting population), $r = 2$, $n = 5$

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Step 3: Work out the multiplier

$1 + \frac{2}{100} = 1 + 0.02 = 1.02$

Step 4: Substitute into the formula

$A = 42{,}000 \times (1.02)^5$

Step 5: Work out the power first

$(1.02)^5 = 1.1040\ldots$

Step 6: Multiply by the starting population

$A = 42{,}000 \times 1.1040\ldots = 46{,}371.3937\ldots$

Step 7: Round to a whole number

$46{,}371.3937\ldots \approx 46{,}371$

We cannot have part of a person, so round to the nearest whole number.

ANSWER **46,371 people**

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Practice questions || Depreciation and other growth and decay

Show all your working. Answers are in the answer key at the back.

Q5.1 A van costs £24,000 and depreciates by 18% per year. Find its value after 3 years. **[3 marks]**

Q5.2 A bike costs £650 and loses 25% of its value each year. How much value has it lost after 2 years? **[3 marks]**

Q5.3 A colony has 8,000 bees. The number falls by 6% each year. Estimate the number of bees after 4 years. **[3 marks]**

Q5.4 A boat is worth £72,000 and depreciates by 9% per year. Work out its value after 5 years. **[3 marks]**

Q5.5 A village has 15,000 people and grows by 4% per year. Estimate the population after 6 years. **[3 marks]**

Q5.6 A television costs £1,500. It depreciates by 35% per year. Find its value after 3 years. **[3 marks]**

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CONCEPT 6

Interest added more than once a year

Some accounts add interest more than once a year: every half year, every quarter (3 months) or every month. The yearly rate is then **shared out** between the periods, and the number of periods goes up.

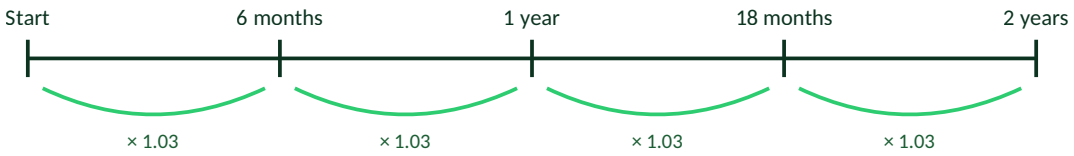
KEY IDEA

$$\text{Rate per period} = \frac{\text{yearly rate}}{\text{number of periods in a year}}$$

Number of periods = years × number of periods in a year

Half yearly	2 periods a year	Quarterly	4 periods a year	Monthly	12 periods a year

Example: 6% per year compounded half yearly for 2 years means 3% every half year, applied 4 times.



Adding interest more often gives slightly **more** money than adding it once a year, because interest starts earning interest sooner.

WATCH OUT

- Divide the **rate** by the number of periods, and multiply the **years** by the number of periods. Do both.
- If the question already gives a monthly rate (for example 0.5% per month), do not divide it again. Just count the months.

Worked examples || Interest added more than once a year

Example 6.1 £2,000 is invested at 6% per year, compounded half yearly, for 3 years. Find the final amount. **[3 marks]**

Step 1: Write down the formula

$$A = P \times \left(1 + \frac{r}{100}\right)^n$$

Step 2: Find the rate for each period

Interest is added every half year, so the yearly rate is shared between 2 periods.

Rate per half year = $\frac{6\%}{2} = 3\%$

Step 3: Find the number of periods

n = 3 years × 2 = 6 periods

Step 4: Work out the multiplier

$1 + \frac{3}{100} = 1 + 0.03 = 1.03$

Step 5: Substitute into the formula

$A = 2{,}000 \times (1.03)^6$

Step 6: Work out the power first

$(1.03)^6 = 1.1940\ldots$

Step 7: Multiply by the starting amount

$A = 2{,}000 \times 1.1940\ldots = 2{,}388.1045\ldots$

Step 8: Round to the nearest penny

$A = £2{,}388.10$ (2 d.p.)

ANSWER **Total = £2,388.10**

Example 6.2 £5,000 is invested at 8% per year, compounded quarterly, for 2 years. Find the final amount. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every quarter, so the yearly rate is shared between 4 periods.

Rate per quarter $= \frac{8\%}{4} = 2\%$

Step 3: Find the number of periods

$n = 2$ years $\times 4 = 8$ periods

Step 4: Work out the multiplier

$1 + \frac{2}{100} = 1 + 0.02 = 1.02$

Step 5: Substitute into the formula

$A = 5{,}000 \times (1.02)^8$

Step 6: Work out the power first

$(1.02)^8 = 1.1716\ldots$

Step 7: Multiply by the starting amount

$A = 5{,}000 \times 1.1716\ldots = 5{,}858.2969\ldots$

Step 8: Round to the nearest penny

$A = £5{,}858.30$ (2 d.p.)

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ANSWER **Total = £5,858.30**

Example 6.3 £1,000 is invested at 12% per year, compounded monthly, for 1 year. Find the final amount. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every month, so the yearly rate is shared between 12 periods.

Rate per month = $\frac{12\%}{12} = 1\%$

Step 3: Find the number of periods

$n = 1 \text{ years} \times 12 = 12 \text{ periods}$

Step 4: Work out the multiplier

$1 + \frac{1}{100} = 1 + 0.01 = 1.01$

Step 5: Substitute into the formula

$A = 1{,}000 \times (1.01)^{12}$

Step 6: Work out the power first

$(1.01)^{12} = 1.1268\ldots$

Step 7: Multiply by the starting amount

$A = 1{,}000 \times 1.1268\ldots = 1{,}126.8250\ldots$

Step 8: Round to the nearest penny

$A = £1{,}126.83$ (2 d.p.)

ANSWER **Total = £1,126.83**

Example 6.4 £3,600 is invested at 4% per year, compounded half yearly, for 5 years. How much interest is earned? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every half year, so the yearly rate is shared between 2 periods.

Rate per half year = $\frac{4\%}{2} = 2\%$

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Step 3: Find the number of periods

n = 5 years × 2 = 10 periods

Step 4: Work out the multiplier

$1 + \frac{2}{100} = 1 + 0.02 = 1.02$

Step 5: Substitute into the formula

$A = 3{,}600 \times (1.02)^{10}$

Step 6: Work out the power first

$(1.02)^{10} = 1.2189\ldots$

Step 7: Multiply by the starting amount

$A = 3{,}600 \times 1.2189\ldots = 4{,}388.3799\ldots$

Step 8: Round to the nearest penny

A = £4,388.38 (2 d.p.)

Step 9: Find the interest

Interest = final amount − original amount

= 4,388.3799… − 3,600 = 788.3799…

= £788.38 (2 d.p.)

ANSWER **Interest = £788.38**

Example 6.5 £800 is saved in an account paying 0.5% per month compound interest. Find the balance after 2 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

The rate is already given per month: 0.5%. Do not divide it again.

Step 3: Find the number of periods

n = 2 years × 12 = 24 periods

Step 4: Work out the multiplier

$1 + \frac{0.5}{100} = 1 + 0.005 = 1.005$

Step 5: Substitute into the formula

$A = 800 \times (1.005)^{24}$

Step 6: Work out the power first

$(1.005)^{24} = 1.1271\ldots$

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Step 7: Multiply by the starting amount

$A = 800 \times 1.1271\ldots = 901.7278\ldots$

Step 8: Round to the nearest penny

$A = £901.73$ (2 d.p.)

ANSWER **Total = £901.73**

Example 6.6 Account A pays 6% per year added once a year. Account B pays 6% per year added half yearly. £10,000 is invested in each for 2 years. Which account gives more, and by how much? **[4 marks]**

Step 1: Yearly compounding

$$\text{Multiplier} = 1 + \frac{6}{100} = 1.06$$

$10{,}000 \times (1.06)^2 = 10{,}000 \times 1.1236 = 11{,}236$

Step 2: Half yearly compounding

$$\text{Rate per half year} = \frac{6\%}{2} = 3\%; \text{ periods} = 2 \times 2 = 4$$

$$\text{Multiplier} = 1 + \frac{3}{100} = 1.03$$

$10{,}000 \times (1.03)^4 = 10{,}000 \times 1.1255\ldots = 11{,}255.0881$

Step 3: Find the difference

$11{,}255.0881 - 11{,}236 = 19.0881$

$= £19.09$ (2 d.p.)

ANSWER **Account B by £19.09**

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Practice questions || Interest added more than once a year

Show all your working. Answers are in the answer key at the back.

Q6.1 £1,500 is invested at 10% per year, compounded half yearly, for 2 years. Find the final amount. **[3 marks]**

Q6.2 £4,000 is invested at 6% per year, compounded quarterly, for 3 years. Find the final amount. **[3 marks]**

Q6.3 £2,400 is invested at 3% per year, compounded half yearly, for 4 years. Find the final amount. **[3 marks]**

Q6.4 £600 is saved at 1.5% per month compound interest for 1 year. Find the balance. **[3 marks]**

Q6.5 £9,000 is invested at 4% per year, compounded quarterly, for 5 years. How much interest is earned? **[3 marks]**

Q6.6 £750 is saved at 0.8% per month compound interest for 3 years. Find the balance. **[3 marks]**

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CONCEPT 7

Working backwards: finding the original amount

Sometimes you are told the amount **after** the interest or depreciation, and asked for the **original** amount. This is reverse percentage working.

Start from the formula with P unknown: $P \times \text{multiplier}^n = A$. To get P on its own, carry the multiplier across the equals sign. When it crosses, **multiplication becomes division**.

KEY IDEA: CARRY ACROSS, THEN THE SIGN CHANGES

$$P \times (\text{multiplier})^n = A \rightarrow P = \frac{A}{(\text{multiplier})^n}$$

WATCH OUT

- The biggest mistake: a price after a 20% rise is £120, so someone takes 20% off £120 and gets £96. That is wrong. 20% of the **new** price is not the same as 20% of the **original**. Always divide by the multiplier.
- Check your answer by multiplying forward again. You should get back to the amount in the question.

Worked examples || Working backwards: finding the original amount

Example 7.1 After 1 year at 5% interest, a savings account holds £1,260. How much was put in? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$$P \times (1 + \frac{r}{100})^n = A$$

Step 2: Work out the multiplier

$$1 + \frac{5}{100} = 1.05$$

Step 3: Substitute

$$P \times 1.05 = 1{,}260$$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$$P = \frac{1{,}260}{1.05}$$

Step 5: Divide

$$P = 1{,}200$$

Step 6: Check

$1{,}200 \times (1.05)^1 = 1{,}260 \approx 1{,}260$ ✓

ANSWER **Original = £1,200.00**

Example 7.2 In a sale, a coat is reduced by 20%. The sale price is £68. What was the original price? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 - \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 - \frac{20}{100} = 0.8$

Step 3: Substitute

$P \times 0.8 = 68$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{68}{0.8}$

Step 5: Divide

$P = 85$

Step 6: Check

$85 \times (0.8)^1 = 68 \approx 68$ ✓

ANSWER **Original = £85.00**

Example 7.3 Money is invested at 4% per year compound interest. After 3 years it is worth £2,249.73. Find the amount invested. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{4}{100} = 1.04$

Step 3: Substitute

$P \times (1.04)^3 = 2{,}249.73$

$P \times 1.124864 = 2{,}249.73$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{2{,}249.73}{1.124864}$

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Step 5: Divide

$P = 2{,}000.0017\ldots$

$P = £2{,}000.00$ (2 d.p.)

Step 6: Check

$2{,}000 \times (1.04)^3 = 2{,}249.728 \approx 2{,}249.73$ ✓

ANSWER **Original = £2,000.00**

Example 7.4 A car depreciates by 10% per year. After 2 years it is worth £12,150. What was it worth when new? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 - \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 - \frac{10}{100} = 0.9$

Step 3: Substitute

$P \times (0.9)^2 = 12{,}150$

$P \times 0.81 = 12{,}150$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{12{,}150}{0.81}$

Step 5: Divide

$P = 15{,}000$

Step 6: Check

$15{,}000 \times (0.9)^2 = 12{,}150 \approx 12{,}150$ ✓

ANSWER **Original = £15,000.00**

Example 7.5 An investment earns 3% per year compound interest. After 5 years it is worth £4,057.46. Find the original investment. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{3}{100} = 1.03$

Step 3: Substitute

$P \times (1.03)^5 = 4{,}057.46$

$P \times 1.1592... = 4{,}057.46$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{4{,}057.46}{1.1592...}$

Step 5: Divide

$P = 3{,}500.0006...$

$P = £3{,}500.00$ (2 d.p.)

Step 6: Check

$3{,}500 \times (1.03)^5 = 4{,}057.4592... \approx 4{,}057.46$ ✓

ANSWER **Original = £3,500.00**

Example 7.6 A fish population falls by 25% each year. After 3 years there are 2,700 fish. How many fish were there at the start? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 - \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 - \frac{25}{100} = 0.75$

Step 3: Substitute

$P \times (0.75)^3 = 2{,}700$

$P \times 0.421875 = 2{,}700$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{2{,}700}{0.421875}$

Step 5: Divide

$P = 6{,}400$

Step 6: Check

$6{,}400 \times (0.75)^3 = 2{,}700 \approx 2{,}700$ ✓

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ANSWER Original = 6,400 fish

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Practice questions || Working backwards: finding the original amount

Show all your working. Answers are in the answer key at the back.

Q7.1 After a pay rise of 8%, a weekly wage is £540. What was the wage before the rise? **[3 marks]**

Q7.2 A sofa is reduced by 15% in a sale. It now costs £714. Find the original price. **[3 marks]**

Q7.3 An account pays 6% per year compound interest. After 2 years it holds £1,123.60. How much was invested? **[3 marks]**

Q7.4 After 3 years at 5% per year compound interest, an investment is worth £3,472.88. Find the original amount. **[3 marks]**

Q7.5 A machine depreciates by 20% per year. After 3 years it is worth £10,240. Find its original value. **[3 marks]**

Q7.6 After 4 years at 2.5% per year compound interest, an account holds £6,622.88. Find the amount invested. **[3 marks]**

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CONCEPT 8

Finding the number of years

Questions such as *"How many years until the money goes over £1,200?"* or *"When will the car be worth less than half its price?"* are answered by **trial**: multiply by the multiplier one year at a time and stop as soon as the target is passed.

METHOD

1. Work out the multiplier.
2. Multiply the starting value by the multiplier, then multiply the answer by the multiplier again, and so on.
3. Record each year in a table.
4. Stop at the **first** year that passes the target. That year is the answer.

Calculator tip: type the starting value and press =. Then type × multiplier and press =. Every time you press = again, the calculator multiplies by the multiplier once more, giving the next year.

WATCH OUT

- The answer is always a **whole** number of years, because interest is only added at the end of each year.
- Show the year just **before** the target as well as the year that passes it; examiners give marks for this.
- "Double" means reach 2 times the starting value. "Half" means fall to 0.5 times the starting value.

Worked examples || Finding the number of years

Example 8.1 £1,000 is invested at 5% per year compound interest. After how many years will it first be worth more than £1,200? **[3 marks]**

Step 1: Work out the multiplier

$$1 + \frac{5}{100} = 1.05$$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £1,200?
1	1,000 × 1.05	£1,050	No
2	(previous) × 1.05	£1,102.5	No
3	(previous) × 1.05	£1,157.625	No
4	(previous) × 1.05	£1,215.50625	Yes ✓

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Step 3: Decide

After 3 years the value is £1,157.625, which has not yet gone above £1,200.

After 4 years the value is £1,215.50625, which is above £1,200.

ANSWER 4 years

Example 8.2 A car worth £16,000 depreciates by 20% per year. After how many years will it first be worth less than half its value? **[3 marks]**

Step 1: Work out the multiplier

$1 - \frac{20}{100} = 0.8$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Below £8,000?
1	16,000 × 0.8	£12,800	No
2	(previous) × 0.8	£10,240	No
3	(previous) × 0.8	£8,192	No
4	(previous) × 0.8	£6,553.6	Yes ✓

Step 3: Decide

After 3 years the value is £8,192, which has not yet gone below £8,000.

After 4 years the value is £6,553.6, which is below £8,000.

ANSWER 4 years

Example 8.3 £5,000 is invested at 8% per year compound interest. How many years does it take to double? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{8}{100} = 1.08$

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Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £10,000?
1	5,000 × 1.08	£5,400	No
2	(previous) × 1.08	£5,832	No
3	(previous) × 1.08	£6,298.56	No
4	(previous) × 1.08	£6,802.4448	No
5	(previous) × 1.08	£7,346.640384	No
6	(previous) × 1.08	£7,934.3716...	No
7	(previous) × 1.08	£8,569.1213...	No
8	(previous) × 1.08	£9,254.6510...	No
9	(previous) × 1.08	£9,995.0231...	No
10	(previous) × 1.08	£10,794.6249...	Yes ✓

Step 3: Decide

After 9 years the value is £9,995.0231..., which has not yet gone above £10,000.

After 10 years the value is £10,794.6249..., which is above £10,000.

ANSWER **10 years**

Example 8.4 A town of 2,000 people grows by 15% per year. After how many years will the population first be more than 3,000? **[3 marks]**

Step 1: Work out the multiplier

$$1 + \frac{15}{100} = 1.15$$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above 3,000?
1	2,000 × 1.15	2,300	No
2	(previous) × 1.15	2,645	No
3	(previous) × 1.15	3,041.75	Yes ✓

Step 3: Decide

After 2 years the value is 2,645, which has not yet gone above 3,000.

After 3 years the value is 3,041.75, which is above 3,000.

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ANSWER 3 years

Example 8.5 £800 is saved at 2.5% per year compound interest. After how many years will it first be more than £900? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{2.5}{100} = 1.025$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £900?
1	800 × 1.025	£820	No
2	(previous) × 1.025	£840.5	No
3	(previous) × 1.025	£861.5125	No
4	(previous) × 1.025	£883.0503…	No
5	(previous) × 1.025	£905.1265…	Yes ✓

Step 3: Decide

After 4 years the value is £883.0503…, which has not yet gone above £900.

After 5 years the value is £905.1265…, which is above £900.

ANSWER 5 years

Example 8.6 A phone costs £1,100 and loses 30% of its value each year. After how many years is it first worth less than £300? **[3 marks]**

Step 1: Work out the multiplier

$1 - \frac{30}{100} = 0.7$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Below £300?
1	1,100 × 0.7	£770	No
2	(previous) × 0.7	£539	No
3	(previous) × 0.7	£377.3	No
4	(previous) × 0.7	£264.11	Yes ✓

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Step 3: Decide

After 3 years the value is £377.3, which has not yet gone below £300.

After 4 years the value is £264.11, which is below £300.

ANSWER 4 years

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Practice questions || Finding the number of years

Show all your working. Answers are in the answer key at the back.

Q8.1 £2,000 is invested at 6% per year compound interest. After how many years will it first be worth more than £2,500? **[3 marks]**

Q8.2 £300 is invested at 10% per year compound interest. After how many years will it first be worth more than £450? **[3 marks]**

Q8.3 A car is worth £9,000 and depreciates by 15% per year. After how many years is it first worth less than £5,000? **[3 marks]**

Q8.4 £4,000 is invested at 4% per year compound interest. After how many years will it first exceed £4,800? **[3 marks]**

Q8.5 A dish has 500 bacteria. The number grows by 40% every hour. After how many hours will there first be more than 2,000 bacteria? **[3 marks]**

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Q8.6 A machine worth £12,000 depreciates by 25% per year. After how many years is it first worth less than £3,000? **[3 marks]**

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CONCEPT 9

Finding the interest rate

Stretch If you know the starting amount, the final amount and the number of years, you can find the **rate**. Use the carry over method, changing the operation each time something crosses the equals sign:

KEY IDEA: UNDO IN REVERSE ORDER

Multiplying	→	crosses over and becomes **dividing**
Power *n*	→	crosses over and becomes the ***n*th root** ($\sqrt{}$ for 2 years, $\sqrt[3]{}$ for 3 years)
Adding 1	→	crosses over and becomes **subtracting** 1
Dividing by 100	→	crosses over and becomes **multiplying** by 100

Shortcut to remember: rate = (multiplier − 1) × 100 for growth, and rate = (1 − multiplier) × 100 for decay.

WATCH OUT

- Find the root of the **whole** fraction {V("A")} ÷ {V("P")}, not of {V("A")} alone.
- A multiplier of 1.05 means 5%, not 105% and not 1.05%.

Worked examples || Finding the interest rate

Example 9.1 £1,000 grows to £1,102.50 in 2 years with compound interest. Find the yearly interest rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$1{,}000 \times (1 + \frac{r}{100})^2 = 1{,}102.5$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^2 = \frac{1{,}102.5}{1{,}000} = 1.1025$

Step 3: Carry the power across as a root

The power 2 crosses the equals sign and becomes a square root.

$1 + \frac{r}{100} = \sqrt{1.1025} = 1.05$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.05 - 1 = 0.05$

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Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.05 \times 100 = 5$

$r = 5$

ANSWER **Rate = 5% per year**

Example 9.2 £2,500 grows to £2,812.16 in 3 years with compound interest. Find the yearly interest rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$2{,}500 \times (1 + \frac{r}{100})^3 = 2{,}812.16$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^3 = \frac{2{,}812.16}{2{,}500} = 1.124864$

Step 3: Carry the power across as a root

The power 3 crosses the equals sign and becomes a cube root.

$1 + \frac{r}{100} = \sqrt[3]{1.124864} = 1.04$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.04 - 1 = 0.04$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.04 \times 100 = 4$

$r = 4$

ANSWER **Rate = 4% per year**

Example 9.3 A car bought for £20,000 is worth £12,800 after 2 years. Find the yearly rate of depreciation. **[4 marks]**

Step 1: Write the formula with the rate unknown

$20{,}000 \times (1 - \frac{r}{100})^2 = 12{,}800$

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Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 - \frac{r}{100})^2 = \frac{12{,}800}{20{,}000} = 0.64$

Step 3: Carry the power across as a root

The power 2 crosses the equals sign and becomes a square root.

$1 - \frac{r}{100} = \sqrt{0.64} = 0.8$

Step 4: Carry the terms across

Carry $\frac{r}{100}$ to the right (it becomes +) and carry 0.8 to the left (it becomes −):

$1 - 0.8 = \frac{r}{100}$

$\frac{r}{100} = 0.2$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.2 \times 100 = 20$

$r = 20$

ANSWER **Rate = 20% per year**

Example 9.4 £600 grows to £675.31 in 4 years with compound interest. Find the yearly rate to 2 decimal places. **[4 marks]**

Step 1: Write the formula with the rate unknown

$600 \times (1 + \frac{r}{100})^4 = 675.31$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^4 = \frac{675.31}{600} = 1.1255\ldots$

Step 3: Carry the power across as a root

The power 4 crosses the equals sign and becomes a 4th root.

$1 + \frac{r}{100} = \sqrt[4]{1.1255\ldots} = 1.0300\ldots$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.0300\ldots - 1 = 0.0300\ldots$

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Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.0300... \times 100 = 3.0001...$

$r = 3.00$ (2 d.p.)

ANSWER **Rate = 3.00% per year**

Example 9.5 £8,000 grows to £9,501.49 in 5 years with compound interest. Find the yearly interest rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$$8{,}000 \times \left(1 + \frac{r}{100}\right)^5 = 9{,}501.49$$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$$\left(1 + \frac{r}{100}\right)^5 = \frac{9{,}501.49}{8{,}000} = 1.1876...$$

Step 3: Carry the power across as a root

The power 5 crosses the equals sign and becomes a 5th root.

$$1 + \frac{r}{100} = \sqrt[5]{1.1876...} = 1.0349...$$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$$\frac{r}{100} = 1.0349... - 1 = 0.0349...$$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.0349... \times 100 = 3.4999...$

$r = 3.50$ (2 d.p.)

ANSWER **Rate = 3.50% per year**

Example 9.6 A town grows from 12,000 to 15,972 people in 3 years. Find the yearly percentage growth. **[4 marks]**

Step 1: Write the formula with the rate unknown

$$12{,}000 \times \left(1 + \frac{r}{100}\right)^3 = 15{,}972$$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^3 = \frac{15{,}972}{12{,}000} = 1.331$

Step 3: Carry the power across as a root

The power 3 crosses the equals sign and becomes a cube root.

$1 + \frac{r}{100} = \sqrt[3]{1.331} = 1.1$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.1 - 1 = 0.1$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.1 \times 100 = 10$

$r = 10$

ANSWER **Rate = 10% per year**

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Practice questions || Finding the interest rate

Show all your working. Answers are in the answer key at the back.

Q9.1 £400 grows to £441 in 2 years with compound interest. Find the yearly interest rate. **[4 marks]**

Q9.2 £1,500 grows to £1,687.30 in 3 years with compound interest. Find the yearly interest rate. **[4 marks]**

Q9.3 A van worth £30,000 is worth £21,870 after 3 years. Find the yearly rate of depreciation. **[4 marks]**

Q9.4 £750 grows to £844.13 in 4 years with compound interest. Find the yearly rate. **[4 marks]**

Q9.5 £5,000 grows to £6,083.26 in 5 years with compound interest. Find the yearly rate. **[4 marks]**

Q9.6 A population grows from 8,000 to 9,261 in 3 years. Find the yearly percentage growth. **[4 marks]**

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EXAM STYLE

Mixed exam practice

These questions mix all the skills from this note, in the style of exam questions. Decide which method each question needs before you start.

M1 Amira has £4,000 to invest for 3 years. Bank A pays 3% per year compound interest. Bank B pays 3.1% per year simple interest. Which bank gives more money, and by how much? **[4 marks]**

M2 A car costs £21,000. It loses 12% of its value in the first year and then 8% of its value each year after that. Work out its value after 3 years. **[3 marks]**

M3 Show that £6,000 invested at 2.5% per year compound interest for 4 years earns £622.88 interest, to the nearest penny. **[4 marks]**

M4 Tom's savings grew from £3,200 to £3,528 in 2 years with compound interest. Find the yearly interest rate. **[4 marks]**

M5 A house is worth £180,000. Its value rises by 4% per year. After how many years will it first be worth more than £220,000? **[3 marks]**

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M6 After 3 years at 6% per year compound interest, Kemi's account holds £2,977.54. How much did she invest? **[3 marks]**

M7 £7,500 is invested at 4.8% per year, compounded monthly, for 2 years. Find the final amount. **[4 marks]**

M8 A nature reserve has 3,000 birds. The number falls by 12% each year. Will the number of birds have fallen below half the starting number after 6 years? Show your working. **[3 marks]**

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REVISION

Formula summary

COMPOUND GROWTH

$$A = P \times (1 + \frac{r}{100})^n$$

Compound interest = $A - P$

COMPOUND DECAY (DEPRECIATION)

$$A = P \times (1 - \frac{r}{100})^n$$

Loss in value = $P - A$

SIMPLE INTEREST

Simple interest = $\frac{P \times r \times n}{100}$

MORE THAN ONCE A YEAR

Rate per period = $\frac{\text{yearly rate}}{\text{periods per year}}$ Number of periods = years × periods per year

WORKING BACKWARDS

Original amount: $P = \frac{A}{(\text{multiplier})^n}$ Rate: multiplier = $\sqrt[n]{\frac{A}{P}}$, then r = (multiplier − 1) × 100

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Which method do I use?

The question says	Use
"simple interest"	Same interest every year: $\frac{P \times r \times n}{100}$
"compound interest", "per annum", "each year"	$P \times (\text{multiplier})^n$
"depreciates", "loses value", "falls by"	Multiplier below 1
"half yearly", "quarterly", "monthly"	Divide the rate, multiply the years
"originally", "before", "how much was invested"	Divide by $(\text{multiplier})^n$
"after how many years", "double", "half"	Trial, one year at a time
"find the rate"	Carry over: divide, root, subtract 1, multiply by 100

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ANSWER KEY

Answer summary

Question	Final answer
Q1.1	**1.09**
Q1.2	**0.78**
Q1.3	**1.005**
Q1.4	**£627.20**
Q1.5	**£41.40**
Q1.6	**£3,056.00**
Q2.1	**£60.00**
Q2.2	**£2,124.00**
Q2.3	**£190.00**
Q2.4	**£5,824.00**
Q2.5	**£122.40**
Q2.6	**£11,500.00**
Q3.1	**£726.00**
Q3.2	**£463.05**
Q3.3	**£2,653.02**
Q3.4	**£199.68**
Q3.5	**£3,374.59**
Q3.6	**£7.71 more**
Q4.1	**£2,951.70**
Q4.2	**£143.29**
Q4.3	**£20,771.76**
Q4.4	**£1,470.72 (I = £490.72)**
Q4.5	**Shown (£8,659.46)**
Q4.6	**£844.10**
Q5.1	**£13,232.83**
Q5.2	**£284.38**
Q5.3	**6,246 bees**
Q5.4	**£44,930.31**
Q5.5	**18,980 people**
Q5.6	**£411.94**
Q6.1	**£1,823.26**

Question	Final answer
Q6.2	**£4,782.47**
Q6.3	**£2,703.58**
Q6.4	**£717.37**
Q6.5	**£1,981.71**
Q6.6	**£999.17**
Q7.1	**£500.00**
Q7.2	**£840.00**
Q7.3	**£1,000.00**
Q7.4	**£3,000.00**
Q7.5	**£20,000.00**
Q7.6	**£6,000.00**
Q8.1	**4 years**
Q8.2	**5 years**
Q8.3	**4 years**
Q8.4	**5 years**
Q8.5	**5 hours**
Q8.6	**5 years**
Q9.1	**5%**
Q9.2	**4.00%**
Q9.3	**10%**
Q9.4	**3.00%**
Q9.5	**4.00%**
Q9.6	**5%**
M1	**Bank B, by £1.09**
M2	**£15,641.47**
M3	**Shown (£622.88)**
M4	**5%**
M5	**6 years**
M6	**£2,500.00**
M7	**£8,254.11**
M8	**Yes (1,393 birds)**

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ANSWER KEY

Full worked solutions

Concept 1 || Percentage multipliers

Q1.1 Write down the multiplier for a increase of 9%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 9% gives 100% + 9% = 109%

Step 2: Change the percentage into a decimal

$$109\% = \frac{109}{100} = 1.09$$

ANSWER **Multiplier = 1.09**

Q1.2 Write down the multiplier for a decrease of 22%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A decrease of 22% gives 100% − 22% = 78%

Step 2: Change the percentage into a decimal

$$78\% = \frac{78}{100} = 0.78$$

ANSWER **Multiplier = 0.78**

Q1.3 Write down the multiplier for a increase of 0.5%. **[1 mark]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 0.5% gives 100% + 0.5% = 100.5%

Step 2: Change the percentage into a decimal

$$100.5\% = \frac{100.5}{100} = 1.005$$

ANSWER **Multiplier = 1.005**

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Q1.4 Increase £560 by 12% using a multiplier. **[2 marks]**

Step 1: Start from 100%

The original amount is always 100%.

A increase of 12% gives 100% + 12% = 112%

Step 2: Change the percentage into a decimal

$112\% = \frac{112}{100} = 1.12$

Step 3: Multiply the original amount by the multiplier

£560 × 1.12 = £627.20

ANSWER **£627.20**

Q1.5 Decrease £45 by 8% using a multiplier. **[2 marks]**

Step 1: Start from 100%

The original amount is always 100%.

A decrease of 8% gives 100% − 8% = 92%

Step 2: Change the percentage into a decimal

$92\% = \frac{92}{100} = 0.92$

Step 3: Multiply the original amount by the multiplier

£45 × 0.92 = £41.40

ANSWER **£41.40**

Q1.6 Decrease £3,200 by 4.5% using a multiplier. **[2 marks]**

Step 1: Start from 100%

The original amount is always 100%.

A decrease of 4.5% gives 100% − 4.5% = 95.5%

Step 2: Change the percentage into a decimal

$95.5\% = \frac{95.5}{100} = 0.955$

Step 3: Multiply the original amount by the multiplier

£3,200 × 0.955 = £3,056.00

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ANSWER £3,056.00

Concept 2 || Simple interest (for comparison)

Q2.1 £300 is invested at 4% per year **simple** interest for 5 years. Work out the interest earned. [2 marks]

Step 1: Find the interest for one year

4% of £300 = $\frac{4}{100} \times 300$

= £12.00

Step 2: Multiply by the number of years

£12.00 × 5 = £60.00

Simple interest is the same every year because it is always worked out on the original amount.

ANSWER **Interest = £60.00**

Q2.2 £1,800 is invested at 6% per year **simple** interest for 3 years. Work out the total amount in the account at the end. [2 marks]

Step 1: Find the interest for one year

6% of £1,800 = $\frac{6}{100} \times 1{,}800$

= £108.00

Step 2: Multiply by the number of years

£108.00 × 3 = £324.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£1,800.00 + £324.00 = £2,124.00

ANSWER **Total = £2,124.00**

Q2.3 £950 is invested at 2.5% per year **simple** interest for 8 years. Work out the interest earned. [2 marks]

Step 1: Find the interest for one year

2.5% of £950 = $\frac{2.5}{100} \times 950$

= £23.75

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Step 2: Multiply by the number of years

£23.75 × 8 = £190.00

Simple interest is the same every year because it is always worked out on the original amount.

ANSWER **Interest = £190.00**

Q2.4 £5,200 is invested at 3% per year **simple** interest for 4 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

3% of £5,200 = $\frac{3}{100}$ × 5,200

= £156.00

Step 2: Multiply by the number of years

£156.00 × 4 = £624.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£5,200.00 + £624.00 = £5,824.00

ANSWER **Total = £5,824.00**

Q2.5 £720 is invested at 8.5% per year **simple** interest for 2 years. Work out the interest earned. **[2 marks]**

Step 1: Find the interest for one year

8.5% of £720 = $\frac{8.5}{100}$ × 720

= £61.20

Step 2: Multiply by the number of years

£61.20 × 2 = £122.40

Simple interest is the same every year because it is always worked out on the original amount.

ANSWER **Interest = £122.40**

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Q2.6 £10,000 is invested at 1.5% per year **simple** interest for 10 years. Work out the total amount in the account at the end. **[2 marks]**

Step 1: Find the interest for one year

1.5% of £10,000 = $\frac{1.5}{100} \times 10{,}000$

= £150.00

Step 2: Multiply by the number of years

£150.00 × 10 = £1,500.00

Simple interest is the same every year because it is always worked out on the original amount.

Step 3: Add the interest to the original amount

£10,000.00 + £1,500.00 = £11,500.00

ANSWER **Total = £11,500.00**

Concept 3 || Compound interest year by year

Q3.1 £600 is invested at 10% per year **compound** interest. Working year by year, find the total amount after 2 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{10}{100} \times 600 = 60$

Balance = 600 + 60 = **660**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{10}{100} \times 660 = 66$

Balance = 660 + 66 = **726**

Step 3: Round to the nearest penny

Total after 2 years = £726



ANSWER **Total = £726.00**

Q3.2 £400 is invested at 5% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{5}{100} \times 400 = 20$

Balance = 400 + 20 = **420**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{5}{100} \times 420 = 21$

Balance = 420 + 21 = **441**

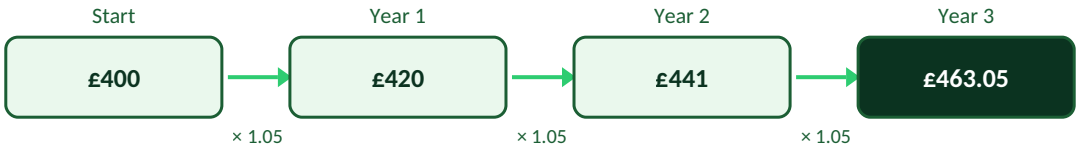
Step 3: Year 3: find the interest and add it on

Interest = $\frac{5}{100} \times 441 = 22.05$

Balance = 441 + 22.05 = **463.05**

Step 4: Round to the nearest penny

Total after 3 years = £463.05



ANSWER **Total = £463.05**

Q3.3 £2,500 is invested at 2% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{2}{100} \times 2{,}500 = 50$

Balance = 2,500 + 50 = **2,550**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{2}{100} \times 2{,}550 = 51$

Balance = 2,550 + 51 = **2,601**

Step 3: Year 3: find the interest and add it on

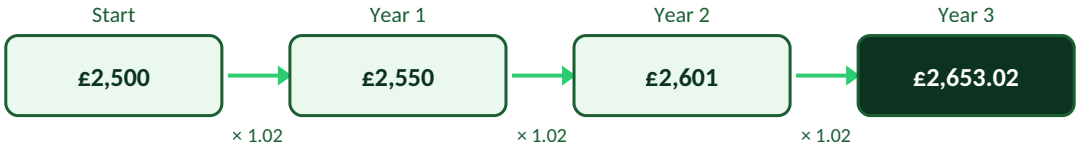
Interest = $\frac{2}{100} \times 2{,}601 = 52.02$

Balance = 2,601 + 52.02 = **2,653.02**

Step 4: Round to the nearest penny

Total after 3 years = £2,653.02

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ANSWER **Total = £2,653.02**

Q3.4 £1,200 is invested at 8% per year **compound** interest. Working year by year, find the total interest earned after 2 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{8}{100} \times 1{,}200 = 96$

Balance = 1,200 + 96 = **1,296**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{8}{100} \times 1{,}296 = 103.68$

Balance = 1,296 + 103.68 = **1,399.68**

Step 3: Round to the nearest penny

Total after 2 years = £1,399.68

Step 4: Find the total interest

Interest = final amount − original amount

= 1,399.68 − 1,200 = 199.68

= £199.68 (2 d.p.)



ANSWER **Interest = £199.68**

Q3.5 £3,000 is invested at 4% per year **compound** interest. Working year by year, find the total amount after 3 years. **[3 marks]**

Step 1: Year 1: find the interest and add it on

Interest = $\frac{4}{100} \times 3{,}000 = 120$

Balance = 3,000 + 120 = **3,120**

Step 2: Year 2: find the interest and add it on

Interest = $\frac{4}{100}$ × 3,120 = 124.80

Balance = 3,120 + 124.80 = **3,244.80**

Step 3: Year 3: find the interest and add it on

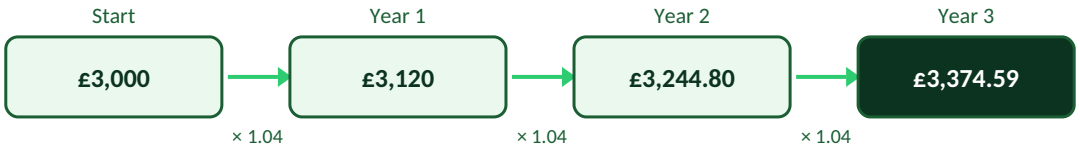
Interest = $\frac{4}{100}$ × 3,244.80 = 129.792

Balance = 3,244.80 + 129.792 = **3,374.592**

Step 4: Round to the nearest penny

Total after 3 years = £3,374.592

= £3,374.59 (2 d.p., to the nearest penny)



ANSWER **Total = £3,374.59**

Q3.6 £700 is invested for 3 years at 6% per year. How much **more** interest is earned with compound interest than with simple interest? **[4 marks]**

Step 1: Work out the simple interest

One year: $\frac{6}{100}$ × 700 = 42

3 years: 42 × 3 = £126.00

Step 2: Work out the compound amount

Multiplier = 1 + $\frac{6}{100}$ = 1.06

700 × $(1.06)^3$ = 700 × 1.191016 = 833.7112

Step 3: Work out the compound interest

833.7112 − 700 = 133.7112

Step 4: Find the difference

133.7112 − 126 = 7.7112

= £7.71 (2 d.p.)

ANSWER **Compound earns £7.71 more**

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Concept 4 || The compound interest formula

Q4.1 £2,400 is invested at 3% per year compound interest for 7 years. Work out the total amount.

[3 marks]

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 2{,}400$ (starting amount), $r = 3$, $n = 7$

Step 3: Work out the multiplier

$1 + \frac{3}{100} = 1 + 0.03 = 1.03$

Step 4: Substitute into the formula

$A = 2{,}400 \times (1.03)^7$

Step 5: Work out the power first

$(1.03)^7 = 1.2298\ldots$

Step 6: Multiply by the starting amount

$A = 2{,}400 \times 1.2298\ldots = 2{,}951.6972\ldots$

Step 7: Round to the nearest penny

$A =$ £2,951.70 (2 d.p.)

ANSWER **Total = £2,951.70**

Q4.2 £600 is invested at 5.5% per year compound interest for 4 years. How much interest is earned?

[3 marks]

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 600$ (starting amount), $r = 5.5$, $n = 4$

Step 3: Work out the multiplier

$1 + \frac{5.5}{100} = 1 + 0.055 = 1.055$

Step 4: Substitute into the formula

$A = 600 \times (1.055)^4$

Step 5: Work out the power first

$(1.055)^4 = 1.2388\ldots$

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Step 6: Multiply by the starting amount

$A = 600 \times 1.2388... = 743.2947...$

Step 7: Round to the nearest penny

$A = £743.29$ (2 d.p.)

Step 8: Find the interest

Interest = final amount − original amount

= 743.2947... − 600 = 143.2947...

= £143.29 (2 d.p.)

ANSWER **Interest = £143.29**

Q4.3 £15,000 is invested at 2.75% per annum compound interest. Find its value after 12 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 15{,}000$ (starting amount), $r = 2.75$, $n = 12$

Step 3: Work out the multiplier

$1 + \frac{2.75}{100} = 1 + 0.0275 = 1.0275$

Step 4: Substitute into the formula

$A = 15{,}000 \times (1.0275)^{12}$

Step 5: Work out the power first

$(1.0275)^{12} = 1.3847...$

Step 6: Multiply by the starting amount

$A = 15{,}000 \times 1.3847... = 20{,}771.7566...$

Step 7: Round to the nearest penny

$A = £20{,}771.76$ (2 d.p.)

ANSWER **Total = £20,771.76**

Q4.4 £980 is invested at 7% per year compound interest for 6 years. Find the final amount and the interest earned. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 980$ (starting amount), $r = 7$, $n = 6$

Step 3: Work out the multiplier

$1 + \frac{7}{100} = 1 + 0.07 = 1.07$

Step 4: Substitute into the formula

$A = 980 \times (1.07)^6$

Step 5: Work out the power first

$(1.07)^6 = 1.5007...$

Step 6: Multiply by the starting amount

$A = 980 \times 1.5007... = 1{,}470.7157...$

Step 7: Round to the nearest penny

$A = £1{,}470.72$ (2 d.p.)

Step 8: Find the interest

Interest = final amount − original amount

$= 1{,}470.7157... - 980 = 490.7157...$

$= £490.72$ (2 d.p.)

ANSWER **Total = £1,470.72; Interest = £490.72**

Q4.5 Show that £8,000 invested at 2% per year compound interest for 4 years grows to £8,659.46

[3 marks]

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 8{,}000$ (starting amount), $r = 2$, $n = 4$

Step 3: Work out the multiplier

$1 + \frac{2}{100} = 1 + 0.02 = 1.02$

Step 4: Substitute into the formula

$A = 8{,}000 \times (1.02)^4$

Step 5: Work out the power first

$(1.02)^4 = 1.0824...$

Step 6: Multiply by the starting amount

$A = 8{,}000 \times 1.0824... = 8{,}659.45728$

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Step 7: Round to the nearest penny

A = £8,659.46 (2 d.p.)

A = **£8,659.46** as required ✓

ANSWER **Shown: £8,659.46 as required ✓**

Q4.6 A grandparent invests £350 for a newborn at 4.5% per year compound interest. How much is it worth after 20 years? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

P = 350 (starting amount), r = 4.5, n = 20

Step 3: Work out the multiplier

$1 + \frac{4.5}{100} = 1 + 0.045 = 1.045$

Step 4: Substitute into the formula

$A = 350 \times (1.045)^{20}$

Step 5: Work out the power first

$(1.045)^{20} = 2.4117...$

Step 6: Multiply by the starting amount

$A = 350 \times 2.4117... = 844.0999...$

Step 7: Round to the nearest penny

A = £844.10 (2 d.p.)

ANSWER **Total = £844.10**

Concept 5 || Depreciation and other growth and decay

Q5.1 A van costs £24,000 and depreciates by 18% per year. Find its value after 3 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

P = 24,000 (starting value), r = 18, n = 3

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Step 3: Work out the multiplier

$1 - \frac{18}{100} = 1 - 0.18 = 0.82$

Step 4: Substitute into the formula

$A = 24{,}000 \times (0.82)^3$

Step 5: Work out the power first

$(0.82)^3 = 0.551368$

Step 6: Multiply by the starting value

$A = 24{,}000 \times 0.551368 = 13{,}232.832$

Step 7: Round to the nearest penny

$A = £13{,}232.83$ (2 d.p.)

ANSWER **Value = £13,232.83**

Q5.2 A bike costs £650 and loses 25% of its value each year. How much value has it lost after 2 years? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 650$ (starting value), $r = 25$, $n = 2$

Step 3: Work out the multiplier

$1 - \frac{25}{100} = 1 - 0.25 = 0.75$

Step 4: Substitute into the formula

$A = 650 \times (0.75)^2$

Step 5: Work out the power first

$(0.75)^2 = 0.5625$

Step 6: Multiply by the starting value

$A = 650 \times 0.5625 = 365.625$

Step 7: Round to the nearest penny

$A = £365.63$ (2 d.p.)

Step 8: Find the loss in value

Loss = starting value − final value

$= 650 - 365.625 = 284.375$

$= £284.38$ (2 d.p.)

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ANSWER **Loss in value = £284.38**

Q5.3 A colony has 8,000 bees. The number falls by 6% each year. Estimate the number of bees after 4 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 8{,}000$ (starting population), $r = 6$, $n = 4$

Step 3: Work out the multiplier

$1 - \frac{6}{100} = 1 - 0.06 = 0.94$

Step 4: Substitute into the formula

$A = 8{,}000 \times (0.94)^4$

Step 5: Work out the power first

$(0.94)^4 = 0.7807\ldots$

Step 6: Multiply by the starting population

$A = 8{,}000 \times 0.7807\ldots = 6{,}245.99168$

Step 7: Round to a whole number

$6{,}245.99168 \approx 6{,}246$

We cannot have part of a bee, so round to the nearest whole number.

ANSWER **6,246 bees**

Q5.4 A boat is worth £72,000 and depreciates by 9% per year. Work out its value after 5 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 72{,}000$ (starting value), $r = 9$, $n = 5$

Step 3: Work out the multiplier

$1 - \frac{9}{100} = 1 - 0.09 = 0.91$

Step 4: Substitute into the formula

$A = 72{,}000 \times (0.91)^5$

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Step 5: Work out the power first

$(0.91)^5 = 0.6240...$

Step 6: Multiply by the starting value

$A = 72{,}000 \times 0.6240... = 44{,}930.3144...$

Step 7: Round to the nearest penny

$A = £44{,}930.31$ (2 d.p.)

ANSWER **Value = £44,930.31**

Q5.5 A village has 15,000 people and grows by 4% per year. Estimate the population after 6 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

$P = 15{,}000$ (starting population), $r = 4$, $n = 6$

Step 3: Work out the multiplier

$1 + \frac{4}{100} = 1 + 0.04 = 1.04$

Step 4: Substitute into the formula

$A = 15{,}000 \times (1.04)^6$

Step 5: Work out the power first

$(1.04)^6 = 1.2653...$

Step 6: Multiply by the starting population

$A = 15{,}000 \times 1.2653... = 18{,}979.7852...$

Step 7: Round to a whole number

$18{,}979.7852... \approx 18{,}980$

We cannot have part of a person, so round to the nearest whole number.

ANSWER **18,980 people**

Q5.6 A television costs £1,500. It depreciates by 35% per year. Find its value after 3 years. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 - \frac{r}{100})^n$

Step 2: Identify the values

$P = 1{,}500$ (starting value), $r = 35$, $n = 3$

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Step 3: Work out the multiplier

$1 - \frac{35}{100} = 1 - 0.35 = 0.65$

Step 4: Substitute into the formula

$A = 1{,}500 \times (0.65)^3$

Step 5: Work out the power first

$(0.65)^3 = 0.274625$

Step 6: Multiply by the starting value

$A = 1{,}500 \times 0.274625 = 411.9375$

Step 7: Round to the nearest penny

$A = £411.94$ (2 d.p.)

ANSWER **Value = £411.94**

Concept 6 || Interest added more than once a year

Q6.1 £1,500 is invested at 10% per year, compounded half yearly, for 2 years. Find the final amount. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every half year, so the yearly rate is shared between 2 periods.

Rate per half year $= \frac{10\%}{2} = 5\%$

Step 3: Find the number of periods

$n = 2 \text{ years} \times 2 = 4 \text{ periods}$

Step 4: Work out the multiplier

$1 + \frac{5}{100} = 1 + 0.05 = 1.05$

Step 5: Substitute into the formula

$A = 1{,}500 \times (1.05)^4$

Step 6: Work out the power first

$(1.05)^4 = 1.2155\ldots$

Step 7: Multiply by the starting amount

$A = 1{,}500 \times 1.2155\ldots = 1{,}823.259375$

Step 8: Round to the nearest penny

$A = £1{,}823.26$ (2 d.p.)

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ANSWER **Total = £1,823.26**

Q6.2 £4,000 is invested at 6% per year, compounded quarterly, for 3 years. Find the final amount.

[3 marks]

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every quarter, so the yearly rate is shared between 4 periods.

Rate per quarter = $\frac{6\%}{4}$ = 1.5%

Step 3: Find the number of periods

n = 3 years × 4 = 12 periods

Step 4: Work out the multiplier

$1 + \frac{1.5}{100} = 1 + 0.015 = 1.015$

Step 5: Substitute into the formula

$A = 4{,}000 \times (1.015)^{12}$

Step 6: Work out the power first

$(1.015)^{12} = 1.1956\ldots$

Step 7: Multiply by the starting amount

$A = 4{,}000 \times 1.1956\ldots = 4{,}782.4726\ldots$

Step 8: Round to the nearest penny

A = £4,782.47 (2 d.p.)

ANSWER **Total = £4,782.47**

Q6.3 £2,400 is invested at 3% per year, compounded half yearly, for 4 years. Find the final amount.

[3 marks]

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every half year, so the yearly rate is shared between 2 periods.

Rate per half year = $\frac{3\%}{2}$ = 1.5%

Step 3: Find the number of periods

n = 4 years × 2 = 8 periods

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Step 4: Work out the multiplier

$1 + \frac{1.5}{100} = 1 + 0.015 = 1.015$

Step 5: Substitute into the formula

$A = 2{,}400 \times (1.015)^8$

Step 6: Work out the power first

$(1.015)^8 = 1.1264\ldots$

Step 7: Multiply by the starting amount

$A = 2{,}400 \times 1.1264\ldots = 2{,}703.5822\ldots$

Step 8: Round to the nearest penny

$A = £2{,}703.58$ (2 d.p.)

ANSWER **Total = £2,703.58**

Q6.4 £600 is saved at 1.5% per month compound interest for 1 year. Find the balance. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

The rate is already given per month: 1.5%. Do not divide it again.

Step 3: Find the number of periods

$n = 1$ years $\times 12 = 12$ periods

Step 4: Work out the multiplier

$1 + \frac{1.5}{100} = 1 + 0.015 = 1.015$

Step 5: Substitute into the formula

$A = 600 \times (1.015)^{12}$

Step 6: Work out the power first

$(1.015)^{12} = 1.1956\ldots$

Step 7: Multiply by the starting amount

$A = 600 \times 1.1956\ldots = 717.3709\ldots$

Step 8: Round to the nearest penny

$A = £717.37$ (2 d.p.)

ANSWER **Total = £717.37**

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Q6.5 £9,000 is invested at 4% per year, compounded quarterly, for 5 years. How much interest is earned? **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every quarter, so the yearly rate is shared between 4 periods.

Rate per quarter = $\frac{4\%}{4} = 1\%$

Step 3: Find the number of periods

$n = 5 \text{ years} \times 4 = 20 \text{ periods}$

Step 4: Work out the multiplier

$1 + \frac{1}{100} = 1 + 0.01 = 1.01$

Step 5: Substitute into the formula

$A = 9{,}000 \times (1.01)^{20}$

Step 6: Work out the power first

$(1.01)^{20} = 1.2201\ldots$

Step 7: Multiply by the starting amount

$A = 9{,}000 \times 1.2201\ldots = 10{,}981.7103\ldots$

Step 8: Round to the nearest penny

$A = £10{,}981.71$ (2 d.p.)

Step 9: Find the interest

Interest = final amount − original amount

= 10,981.7103… − 9,000 = 1,981.7103…

= £1,981.71 (2 d.p.)

ANSWER **Interest = £1,981.71**

Q6.6 £750 is saved at 0.8% per month compound interest for 3 years. Find the balance. **[3 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

The rate is already given per month: 0.8%. Do not divide it again.

Step 3: Find the number of periods

$n = 3 \text{ years} \times 12 = 36 \text{ periods}$

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Step 4: Work out the multiplier

$1 + \frac{0.8}{100} = 1 + 0.008 = 1.008$

Step 5: Substitute into the formula

$A = 750 \times (1.008)^{36}$

Step 6: Work out the power first

$(1.008)^{36} = 1.3322\ldots$

Step 7: Multiply by the starting amount

$A = 750 \times 1.3322\ldots = 999.1723\ldots$

Step 8: Round to the nearest penny

$A = £999.17$ (2 d.p.)

ANSWER **Total = £999.17**

Concept 7 || Working backwards: finding the original amount

Q7.1 After a pay rise of 8%, a weekly wage is £540. What was the wage before the rise? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{8}{100} = 1.08$

Step 3: Substitute

$P \times 1.08 = 540$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{540}{1.08}$

Step 5: Divide

$P = 500$

Step 6: Check

$500 \times (1.08)^1 = 540 \approx 540$ ✓

ANSWER **Original = £500.00**

Q7.2 A sofa is reduced by 15% in a sale. It now costs £714. Find the original price. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 - \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 - \frac{15}{100} = 0.85$

Step 3: Substitute

$P \times 0.85 = 714$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{714}{0.85}$

Step 5: Divide

$P = 840$

Step 6: Check

$840 \times (0.85)^1 = 714 \approx 714$ ✓

ANSWER **Original = £840.00**

Q7.3 An account pays 6% per year compound interest. After 2 years it holds £1,123.60. How much was invested? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{6}{100} = 1.06$

Step 3: Substitute

$P \times (1.06)^2 = 1{,}123.6$

$P \times 1.1236 = 1{,}123.6$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{1{,}123.6}{1.1236}$

Step 5: Divide

$P = 1{,}000$

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Step 6: Check

$1{,}000 \times (1.06)^2 = 1{,}123.6 \approx 1{,}123.6$ ✓

ANSWER **Original = £1,000.00**

Q7.4 After 3 years at 5% per year compound interest, an investment is worth £3,472.88. Find the original amount. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{5}{100} = 1.05$

Step 3: Substitute

$P \times (1.05)^3 = 3{,}472.88$

$P \times 1.157625 = 3{,}472.88$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{3{,}472.88}{1.157625}$

Step 5: Divide

$P = 3{,}000.0043\ldots$

$P =$ £3,000.00 (2 d.p.)

Step 6: Check

$3{,}000 \times (1.05)^3 = 3{,}472.875 \approx 3{,}472.88$ ✓

ANSWER **Original = £3,000.00**

Q7.5 A machine depreciates by 20% per year. After 3 years it is worth £10,240. Find its original value. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 - \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 - \frac{20}{100} = 0.8$

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Step 3: Substitute

$P \times (0.8)^3 = 10{,}240$

$P \times 0.512 = 10{,}240$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{10{,}240}{0.512}$

Step 5: Divide

$P = 20{,}000$

Step 6: Check

$20{,}000 \times (0.8)^3 = 10{,}240 \approx 10{,}240$ ✓

ANSWER **Original = £20,000.00**

Q7.6 After 4 years at 2.5% per year compound interest, an account holds £6,622.88. Find the amount invested. **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{2.5}{100} = 1.025$

Step 3: Substitute

$P \times (1.025)^4 = 6{,}622.88$

$P \times 1.1038... = 6{,}622.88$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{6{,}622.88}{1.1038...}$

Step 5: Divide

$P = 6{,}000.0024...$

$P =$ £6,000.00 (2 d.p.)

Step 6: Check

$6{,}000 \times (1.025)^4 = 6{,}622.8773... \approx 6{,}622.88$ ✓

ANSWER **Original = £6,000.00**

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Concept 8 || Finding the number of years

Q8.1 £2,000 is invested at 6% per year compound interest. After how many years will it first be worth more than £2,500? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{6}{100} = 1.06$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £2,500?
1	2,000 × 1.06	£2,120	No
2	(previous) × 1.06	£2,247.2	No
3	(previous) × 1.06	£2,382.032	No
4	(previous) × 1.06	£2,524.95392	Yes ✓

Step 3: Decide

After 3 years the value is £2,382.032, which has not yet gone above £2,500.

After 4 years the value is £2,524.95392, which is above £2,500.

ANSWER 4 years

Q8.2 £300 is invested at 10% per year compound interest. After how many years will it first be worth more than £450? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{10}{100} = 1.1$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £450?
1	300 × 1.1	£330	No
2	(previous) × 1.1	£363	No
3	(previous) × 1.1	£399.3	No
4	(previous) × 1.1	£439.23	No
5	(previous) × 1.1	£483.153	Yes ✓

Step 3: Decide

After 4 years the value is £439.23, which has not yet gone above £450.

After 5 years the value is £483.153, which is above £450.

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ANSWER 5 years

Q8.3 A car is worth £9,000 and depreciates by 15% per year. After how many years is it first worth less than £5,000? **[3 marks]**

Step 1: Work out the multiplier

$1 - \frac{15}{100} = 0.85$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Below £5,000?
1	9,000 × 0.85	£7,650	No
2	(previous) × 0.85	£6,502.5	No
3	(previous) × 0.85	£5,527.125	No
4	(previous) × 0.85	£4,698.05625	Yes ✓

Step 3: Decide

After 3 years the value is £5,527.125, which has not yet gone below £5,000.

After 4 years the value is £4,698.05625, which is below £5,000.

ANSWER 4 years

Q8.4 £4,000 is invested at 4% per year compound interest. After how many years will it first exceed £4,800? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{4}{100} = 1.04$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £4,800?
1	4,000 × 1.04	£4,160	No
2	(previous) × 1.04	£4,326.4	No
3	(previous) × 1.04	£4,499.456	No
4	(previous) × 1.04	£4,679.43424	No
5	(previous) × 1.04	£4,866.6116…	Yes ✓

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Step 3: Decide

After 4 years the value is £4,679.43424, which has not yet gone above £4,800.

After 5 years the value is £4,866.6116..., which is above £4,800.

ANSWER 5 years

Q8.5 A dish has 500 bacteria. The number grows by 40% every hour. After how many hours will there first be more than 2,000 bacteria? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{40}{100} = 1.4$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Hour	Working	Value	Above 2,000?
1	500 × 1.4	700	No
2	(previous) × 1.4	980	No
3	(previous) × 1.4	1,372	No
4	(previous) × 1.4	1,920.8	No
5	(previous) × 1.4	2,689.12	Yes ✔

Step 3: Decide

After 4 hours the value is 1,920.8, which has not yet gone above 2,000.

After 5 hours the value is 2,689.12, which is above 2,000.

ANSWER 5 hours

Q8.6 A machine worth £12,000 depreciates by 25% per year. After how many years is it first worth less than £3,000? **[3 marks]**

Step 1: Work out the multiplier

$1 - \frac{25}{100} = 0.75$

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Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Below £3,000?
1	12,000 × 0.75	£9,000	No
2	(previous) × 0.75	£6,750	No
3	(previous) × 0.75	£5,062.5	No
4	(previous) × 0.75	£3,796.875	No
5	(previous) × 0.75	£2,847.65625	Yes ✓

Step 3: Decide

After 4 years the value is £3,796.875, which has not yet gone below £3,000.

After 5 years the value is £2,847.65625, which is below £3,000.

ANSWER 5 years

Concept 9 || Finding the interest rate

Q9.1 £400 grows to £441 in 2 years with compound interest. Find the yearly interest rate. [4 marks]

Step 1: Write the formula with the rate unknown

$400 \times (1 + \frac{r}{100})^2 = 441$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^2 = \frac{441}{400} = 1.1025$

Step 3: Carry the power across as a root

The power 2 crosses the equals sign and becomes a square root.

$1 + \frac{r}{100} = \sqrt{1.1025} = 1.05$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.05 - 1 = 0.05$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.05 \times 100 = 5$

$r = 5$

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ANSWER **Rate = 5% per year**

Q9.2 £1,500 grows to £1,687.30 in 3 years with compound interest. Find the yearly interest rate.

[4 marks]

Step 1: Write the formula with the rate unknown

$1{,}500 \times (1 + \frac{r}{100})^3 = 1{,}687.3$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^3 = \frac{1{,}687.3}{1{,}500} = 1.1248\ldots$

Step 3: Carry the power across as a root

The power 3 crosses the equals sign and becomes a cube root.

$1 + \frac{r}{100} = \sqrt[3]{1.1248\ldots} = 1.0400\ldots$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.0400\ldots - 1 = 0.0400\ldots$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.0400\ldots \times 100 = 4.0000\ldots$

$r = 4.00$ (2 d.p.)

ANSWER **Rate = 4.00% per year**

Q9.3 A van worth £30,000 is worth £21,870 after 3 years. Find the yearly rate of depreciation.

[4 marks]

Step 1: Write the formula with the rate unknown

$30{,}000 \times (1 - \frac{r}{100})^3 = 21{,}870$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 - \frac{r}{100})^3 = \frac{21{,}870}{30{,}000} = 0.729$

Step 3: Carry the power across as a root

The power 3 crosses the equals sign and becomes a cube root.

$1 - \frac{r}{100} = \sqrt[3]{0.729} = 0.9$

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Step 4: Carry the terms across

Carry $\frac{r}{100}$ to the right (it becomes +) and carry 0.9 to the left (it becomes −):

$1 - 0.9 = \frac{r}{100}$

$\frac{r}{100} = 0.1$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.1 \times 100 = 10$

$r = 10$

ANSWER **Rate = 10% per year**

Q9.4 £750 grows to £844.13 in 4 years with compound interest. Find the yearly rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$750 \times (1 + \frac{r}{100})^4 = 844.13$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^4 = \frac{844.13}{750} = 1.1255...$

Step 3: Carry the power across as a root

The power 4 crosses the equals sign and becomes a 4th root.

$1 + \frac{r}{100} = \sqrt[4]{1.1255...} = 1.0299...$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.0299... - 1 = 0.0299...$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.0299... \times 100 = 2.9999...$

$r = 3.00$ (2 d.p.)

ANSWER **Rate = 3.00% per year**

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Q9.5 £5,000 grows to £6,083.26 in 5 years with compound interest. Find the yearly rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$5{,}000 \times (1 + \frac{r}{100})^5 = 6{,}083.26$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^5 = \frac{6{,}083.26}{5{,}000} = 1.216652$

Step 3: Carry the power across as a root

The power 5 crosses the equals sign and becomes a 5th root.

$1 + \frac{r}{100} = \sqrt[5]{1.216652} = 1.0399\ldots$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.0399\ldots - 1 = 0.0399\ldots$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.0399\ldots \times 100 = 3.9999\ldots$

$r = 4.00$ (2 d.p.)

ANSWER **Rate = 4.00% per year**

Q9.6 A population grows from 8,000 to 9,261 in 3 years. Find the yearly percentage growth. **[4 marks]**

Step 1: Write the formula with the rate unknown

$8{,}000 \times (1 + \frac{r}{100})^3 = 9{,}261$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^3 = \frac{9{,}261}{8{,}000} = 1.157625$

Step 3: Carry the power across as a root

The power 3 crosses the equals sign and becomes a cube root.

$1 + \frac{r}{100} = \sqrt[3]{1.157625} = 1.05$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.05 - 1 = 0.05$

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Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.05 \times 100 = 5$

$r = 5$

ANSWER Rate = 5% per year

Mixed exam practice

M1 Amira has £4,000 to invest for 3 years. Bank A pays 3% per year compound interest. Bank B pays 3.1% per year simple interest. Which bank gives more money, and by how much? **[4 marks]**

Step 1: Bank A: compound interest

Multiplier = $1 + \frac{3}{100} = 1.03$

$4000 \times (1.03)^3 = 4000 \times 1.092727 = 4,370.908$

= £4,370.91 (2 d.p.)

Step 2: Bank B: simple interest

One year: $\frac{3.1}{100} \times 4000 = 124$

3 years: $124 \times 3 = 372$

Total = 4000 + 372 = £4,372.00

Step 3: Compare

4,372 − 4,370.908 = 1.092

= £1.09 (2 d.p.)

Bank B gives more, even though it uses simple interest, because its rate is higher.

ANSWER Bank B, by £1.09

M2 A car costs £21,000. It loses 12% of its value in the first year and then 8% of its value each year after that. Work out its value after 3 years. **[3 marks]**

Step 1: Year 1: depreciation of 12%

Multiplier = $1 - \frac{12}{100} = 0.88$

$21000 \times 0.88 = 18,480$

Step 2: Years 2 and 3: depreciation of 8% each year

Multiplier = $1 - \frac{8}{100} = 0.92$

$18,480 \times (0.92)^2 = 18,480 \times 0.8464 = 15,641.472$

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Step 3: Round to the nearest penny

= £15,641.47 (2 d.p.)

ANSWER **Value = £15,641.47**

M3 Show that £6,000 invested at 2.5% per year compound interest for 4 years earns £622.88 interest, to the nearest penny. **[4 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Identify the values

P = 6,000 (starting amount), r = 2.5, n = 4

Step 3: Work out the multiplier

$1 + \frac{2.5}{100} = 1 + 0.025 = 1.025$

Step 4: Substitute into the formula

$A = 6{,}000 \times (1.025)^4$

Step 5: Work out the power first

$(1.025)^4 = 1.1038...$

Step 6: Multiply by the starting amount

$A = 6{,}000 \times 1.1038... = 6{,}622.8773...$

Step 7: Round to the nearest penny

A = £6,622.88 (2 d.p.)

Step 8: Find the interest

Interest = 6,622.8773... − 6,000 = 622.8773...

= £622.88 (2 d.p.)

Interest = **£622.88** as required ✓

ANSWER **Shown: £622.88 as required ✓**

M4 Tom's savings grew from £3,200 to £3,528 in 2 years with compound interest. Find the yearly interest rate. **[4 marks]**

Step 1: Write the formula with the rate unknown

$3{,}200 \times (1 + \frac{r}{100})^2 = 3{,}528$

Step 2: Carry the starting value across the equals sign

Multiplying on the left becomes dividing on the right.

$(1 + \frac{r}{100})^2 = \frac{3,528}{3,200} = 1.1025$

Step 3: Carry the power across as a root

The power 2 crosses the equals sign and becomes a square root.

$1 + \frac{r}{100} = \sqrt{1.1025} = 1.05$

Step 4: Carry the 1 across the equals sign

+1 on the left becomes −1 on the right.

$\frac{r}{100} = 1.05 - 1 = 0.05$

Step 5: Carry the 100 across the equals sign

Dividing on the left becomes multiplying on the right.

$r = 0.05 \times 100 = 5$

$r = 5$

ANSWER **Rate = 5% per year**

M5 A house is worth £180,000. Its value rises by 4% per year. After how many years will it first be worth more than £220,000? **[3 marks]**

Step 1: Work out the multiplier

$1 + \frac{4}{100} = 1.04$

Step 2: Multiply one period at a time

Keep multiplying the previous value by the multiplier until the target is passed.

Year	Working	Value	Above £220,000?
1	180,000 × 1.04	£187,200	No
2	(previous) × 1.04	£194,688	No
3	(previous) × 1.04	£202,475.52	No
4	(previous) × 1.04	£210,574.5408	No
5	(previous) × 1.04	£218,997.522432	No
6	(previous) × 1.04	£227,757.4233…	Yes ✓

Step 3: Decide

After 5 years the value is £218,997.522432, which has not yet gone above £220,000.

After 6 years the value is £227,757.4233…, which is above £220,000.

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ANSWER 6 years

M6 After 3 years at 6% per year compound interest, Kemi's account holds £2,977.54. How much did she invest? **[3 marks]**

Step 1: Write the formula with the original amount unknown

$P \times (1 + \frac{r}{100})^n = A$

Step 2: Work out the multiplier

$1 + \frac{6}{100} = 1.06$

Step 3: Substitute

$P \times (1.06)^3 = 2{,}977.54$

$P \times 1.191016 = 2{,}977.54$

Step 4: Carry the multiplier across the equals sign

Multiplying on the left becomes dividing on the right.

$P = \frac{2{,}977.54}{1.191016}$

Step 5: Divide

$P = 2{,}500$

Step 6: Check

$2{,}500 \times (1.06)^3 = 2{,}977.54 \approx 2{,}977.54$ ✓

ANSWER Original = £2,500.00

M7 £7,500 is invested at 4.8% per year, compounded monthly, for 2 years. Find the final amount. **[4 marks]**

Step 1: Write down the formula

$A = P \times (1 + \frac{r}{100})^n$

Step 2: Find the rate for each period

Interest is added every month, so the yearly rate is shared between 12 periods.

Rate per month = $\frac{4.8\%}{12} = 0.4\%$

Step 3: Find the number of periods

$n = 2$ years $\times 12 = 24$ periods

Step 4: Work out the multiplier

$1 + \frac{0.4}{100} = 1 + 0.004 = 1.004$

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Step 5: Substitute into the formula

$A = 7{,}500 \times (1.004)^{24}$

Step 6: Work out the power first

$(1.004)^{24} = 1.1005\ldots$

Step 7: Multiply by the starting amount

$A = 7{,}500 \times 1.1005\ldots = 8{,}254.1122\ldots$

Step 8: Round to the nearest penny

$A = £8{,}254.11$ (2 d.p.)

ANSWER **Total = £8,254.11**

M8 A nature reserve has 3,000 birds. The number falls by 12% each year. Will the number of birds have fallen below half the starting number after 6 years? Show your working. **[3 marks]**

Step 1: Work out the multiplier

$1 - \frac{12}{100} = 0.88$

Step 2: Substitute into the formula

$A = 3000 \times (0.88)^6$

$(0.88)^6 = 0.4644\ldots$

$A = 3000 \times 0.4644\ldots = 1{,}393.2122\ldots$

$\approx 1{,}393$ birds (nearest whole number)

Step 3: Compare with half the starting population

Half of $3000 = \frac{3000}{2} = 1500$

$1{,}393 < 1500$, so the population **has** fallen below half.

ANSWER **Yes: about 1,393 birds, which is below 1,500**

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